Quiz # 10– Math 2163, Calculus III – Nov. 9, 2007

Show all your work neatly and concisely, and indicate your final answer clearly.

1. Find the area of the part of the plane

$$2x + 5y + z = 10$$

that lies inside the cylinder $x^2 + y^2 = 9$.

Solution: The formula for the surface area is

$$\iint_D \sqrt{f_x^2 + f_y^2 + 1}\, dA$$

where D is given by the cylinder that the surface lies in. In this problem, the easiest way to describe D, the disk inside $x^2 + y^2 = 9$, is to use polar coordinates:

$$D = \{(r, \theta) \,|\, 0 \le \theta \le 2\pi,\ 0 \le r \le 3\}.$$

The surface is given by $2x + 5y + z = 10$, which means

$$z = f(x, y) = 10 - 2x - 5y,$$
$$f_x = -2, \qquad f_y = -5.$$

So the area is

$$\begin{aligned}
\iint_D \sqrt{f_x^2 + f_y^2 + 1}\, dA &= \iint_D \sqrt{(-2)^2 + (-5)^2 + 1}\, dA \\
&= \int_0^{2\pi} \int_0^3 \sqrt{30} r\, dr\, d\theta \\
&= \int_0^{2\pi} \frac{\sqrt{30}}{2} r^2 \Big|_{r=0}^{r=3} d\theta \\
&= \int_0^{2\pi} \frac{9\sqrt{30}}{2}\, d\theta \\
&= \frac{9\sqrt{30}}{2} \theta \Big|_{\theta=0}^{\theta=2\pi} = \frac{9\sqrt{30}}{2} 2\pi = 9\sqrt{30}\pi.
\end{aligned}$$