

## Math 2163, Practice Exam III

1. Sketch the 2-D regions:

- (a)  $D = \{(x, y) \mid 0 \leq y \leq 2, 0 \leq x \leq 2 - y\}$ ,
- (b)  $D = \{(x, y) \mid -2 \leq x \leq 2, -x^2 - 1 \leq y \leq \sqrt{4 - x^2}\}$
- (c)  $D = \{(r, \theta) \mid 0 \leq \theta \leq 2\pi, 1 \leq r \leq 6\}$ ,
- (d)  $D = \{(r, \theta) \mid -\pi/4 \leq \theta \leq \pi/4, 0 \leq r \leq \cos 2\theta\}$ .

2. Sketch the 3-D regions:

- (a)  $D = \{(x, y, z) \mid -1 \leq y \leq 1, y^2 \leq x \leq 1, 0 \leq z \leq x\}$ ,
- (b)  $D = \{(x, y, z) \mid 1 \leq x \leq 2, 0 \leq y \leq \sqrt{4 - x^2}, 0 \leq z \leq \sqrt{1 - x^2/4 - y^2/4}\}$ ,
- (c)  $D = \{(r, \theta, z) \mid 0 \leq \theta \leq \pi/2, 0 \leq r \leq 1, 1 - r^2 \leq z \leq 1\}$ ,
- (d)  $D = \{(\rho, \theta, \phi) \mid 1 \leq \rho \leq 2, \pi/2 \leq \theta \leq \pi, 0 \leq \phi \leq \pi\}$ .

3. Evaluate iterated integrals:

- (a)  $\int_0^1 \int_0^1 ye^{xy} dx dy$ ,
- (b)  $\int_0^1 \int_0^y \int_x^1 6xyz dz dx dy$ ,
- (c)  $\int_0^{\pi/2} \int_0^{\sin 2\theta} r dr d\theta$ ,
- (d)  $\int_0^{\pi/2} \int_0^{\pi/2} \int_1^2 \rho^2 \sin \phi d\rho d\phi d\theta$ .

4. Find the area of the region enclosed by the curve  $r = 4 + 3 \cos \theta$ .

5. Find the area of the part of the surface  $z = x^2 + y^2$  lies inside the circle  $x^2 + y^2 = 9$ .

6. Evaluate the volume bounded by paraboloids  $z = 3x^2 + 3y^2$  and  $z = 4 - x^2 - y^2$ .

7. Find the mass and center of the mass of the solid tetrahedron with vertices  $(0, 0, 0)$ ,  $(1, 0, 0)$ ,  $(0, 2, 0)$ ,  $(0, 0, 3)$ , with density function  $\rho(x, y, z) = x^2 + y^2 + z^2$ .

8. Evaluate the triple integrals

- (a)  $\iiint_E xy dV$ , where  $E = \{(x, y, z) \mid 0 \leq x \leq 3, 0 \leq y \leq x, 0 \leq z \leq x + y\}$ ,
- (b)  $\iiint_E yz dV$ , where  $E$  is bounded by  $z = 0$ ,  $z = y$  and lies inside the cylinder  $x^2 + y^2 = 4$ ,  $y \geq 0$ ,
- (c)  $\iiint_E z^3 \sqrt{x^2 + y^2 + z^2} dV$  where  $E$  is the hemisphere that lies above the  $xy$ -plane and has center the origin and radius 1.