Math 2163, Practice Exam II

1. Find the directional derivative of the functions at the given point in the given direction:
   - $f(x,y) = s^2 e^t$, $(2,0)$, $\mathbf{v} = \mathbf{i} + \mathbf{j}$,
   - $f(x,y) = x^2y^3 - y^4$, $(2,1)$, $\theta = \pi/4$,
   - $f(x,y,z) = x/(y+z)$, $(4,1,1)$, $\mathbf{v} =< 1,2,3 >$.

2. Find the maximum rate of change of f at the given point:
   - $f(x,y) = y^2/x$, $(2,4)$,
   - $f(x,y,z) = x^4y^3z^2$, $(1,-1,1)$.

3. Find and classify all critical points of
   - $f(x,y) = (x^2+y^2)e^{y^2-x^2}$,
   - $f(x,y) = 2x^3 + xy^2 + 5x^2 + y^2$.

4. Find the absolute maximum and minimum of f on D:

$$f(x,y) = 4x + 6y - x^2 - y^2, \quad D = [0,4] \times [0,5].$$

5. Use Lagrange multipliers to find the shortest distance from the point $(8,10,8)$ to the plane $8x - 10y + 4z = 16$.

6. Use Lagrange multipliers to find the dimension of the rectangular box with largest volume if the total surface area is given as $150\ cm^2$.

7. Find an approximation for the integral

$$\iint_R (4x - 5y^2)\, dA, \qquad R = [0,8] \times [04]$$

   by a double Riemann sum with $m = n = 2$ and the sample point in upper right corner.

8. Calculate the double integrals:
   - $\int_0^6 \int_0^{10} \sqrt{(x+y)}\, dx\, dy$,
   - $\iint_D \frac{3y}{x^2+1} dA$, $D = \{0 \le x \le 9,\ 0 \le y \le \sqrt{x}\}$,
   - $\int_1^2 \int_0^1 \frac{x}{x^2+y^2}\, dy\, dx$,
   - $\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3+1}\, dx\, dy$.