Math 2163, Practice Exam I, Solution

1. (a) $|\mathbf{a}| = \sqrt{14}$, $3\mathbf{a} + 4\mathbf{b} =< 15, 7, 34 >$, $\mathbf{a} \cdot \mathbf{b} = -3$,$\mathbf{a} \times \mathbf{b} =< 5, 33, -9 >$;
   (b) $|\mathbf{a}| = \sqrt{6}$, $3\mathbf{a} + 4\mathbf{b} =< 3, -2, 11 >$, $\mathbf{a} \cdot \mathbf{b} = 0$,$\mathbf{a} \times \mathbf{b} =< -5, -2, 1 >$.
2. $\frac{12}{13}\mathbf{i} - \frac{5}{13}\mathbf{j}$.
3. (a) $\theta = \arccos \frac{5}{21}$, skew;
   (b) $\theta = \arccos \frac{3}{\sqrt{84}}$, skew;
   (c) $\theta = \arccos 0 = \pi/2$, orthogonal;
   (d) $\theta = \arccos -1 = \pi$, parallel;
   (e) $\theta = \arccos 0 = \pi/2$, orthogonal.
4. The direction cosines are $< \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}, \frac{-1}{\sqrt{6}} >$.
5. The scalar projection is $-1/\sqrt{34}$ and the vector projection is $-\frac{1}{34} < 3, 3, 4 >$.
6. $< \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}} >$ and $< \frac{-1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}} >$.
7. 4.
8. (a) $\frac{x-1}{2} = \frac{y-0}{-4} = \frac{z+3}{5}$;
   (b) $\frac{x+2}{1} = \frac{y-4}{1} = \frac{z-1}{-1}$;
   (c) $\frac{x-1}{1} = \frac{y-1}{-2} = \frac{z-3}{-3}$;
   (d) $\frac{x-2}{1} = \frac{y-1}{1} = \frac{z-0}{-2}$;
   (e) $\frac{x-1}{1} = \frac{z+1}{-1}$, $y = 1$.
9. (a) $(x-4) + (y-1) + 2(z-2) = 0$;
   (b) $(x-4) - (y-1) + (z-2) = 0$;
   (c) $13x - 17y - 7z = 42$;
   (d) $5x - y + 3z = 2$;
10. (a) $\{(x, y) \,|\, y \geq x \text{ and } x + y > 0\}$;
    (b) $\{(x, y) \,|\, x^2 + 16y^2 \leq 16\}$;
    (c) $\{(x, y) \,|\, y \neq x^2\}$;
11. (a) Take limits along the x-axis and the y-axis will give different values;
    (b) Take limits along the x-axis and the line $y = x$ will give different values;
    (c) For this problem, you can not take the path $y = x$, so try taking limits along the x-axis and $y = 2x$ and you will have different values.
12. (a) $f_x = yx^{y-1}$, $f_y = (\ln x)x^y$;

(b) $f_x = \frac{y^4 - x^2y^2}{(x^2+y^2)^2}$, $f_y = \frac{2x^3y}{(x^2+y^2)^2}$;

(c) $f_x = -\cos x$, $f_y = \cos y$;

(d) $f_x = 2xe^{yz}$, $f_y = x^2ze^{yz}$, $f_z = x^2ye^{yz}$;

(e) $f_x = \tan(xy) + xy\sec^2(xy)$, $f_y = x^2\sec^2(xy)$, $f_z = 0$;

(f) By using chain rule, $\partial z/\partial x = -\frac{y}{x^2}\cos\frac{y}{x}$, $\partial z/\partial y = \frac{1}{x}\cos\frac{y}{x}$;

(g) By using chain rule, $dz/dt = -2t$;

(h) There is a typo, the correct equation should be $w = xy + yz + zx$. The solution is $\partial w/\partial s = te^{st} + t^3 + st^2e^{st} + t^3e^{st}$, $\partial w/\partial t = se^{st} + s^2te^{st} + st^2e^{st} + 3st^2 + 2te^{st}$;

(i) By using implicit differentiation, $\partial z/\partial x = -\frac{y^2+2xz}{x^2+2yz}$, $\partial z/\partial y = -\frac{z^2+2xy}{x^2+2yz}$,

(j) By using implicit differentiation, $\partial z/\partial x = -z/x$, $\partial z/\partial y = -z/y$.

13. (a) $-x\sin y$;

(b) $(2x + 4x^2y^2)e^{xy^2}$;

(c) $f_{rss} = -2/s^2$, $f_{rst} = 0$;

14. $dz = e^s \sin t\, ds + e^s \cos t\, dt$.

15. $z = 2x + y - 1$.

16. The linear approximation is $z = x + y$ and the approximate to $f(1.1, -0.1)$ is 1.0.