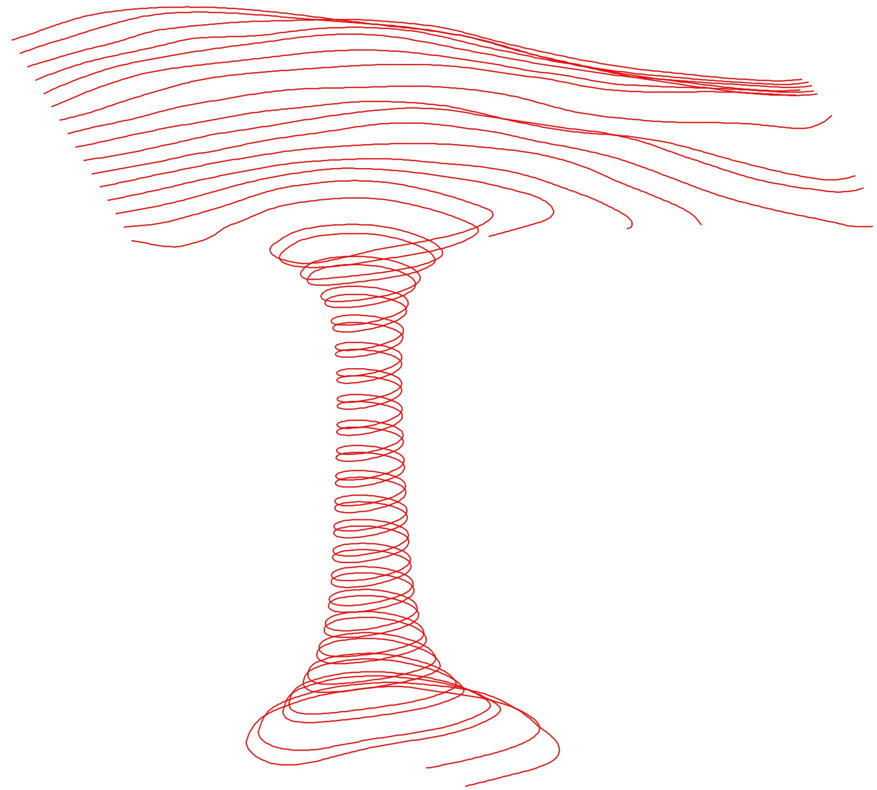
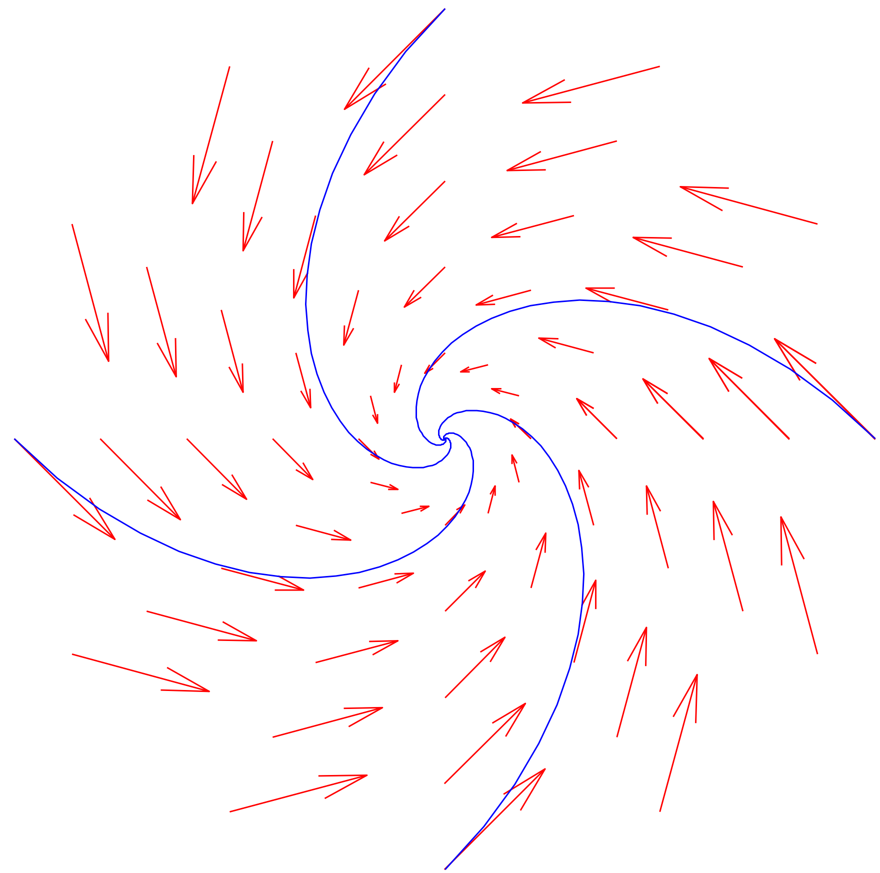
Vector fields

Math 2163

Tornado



Hurricane

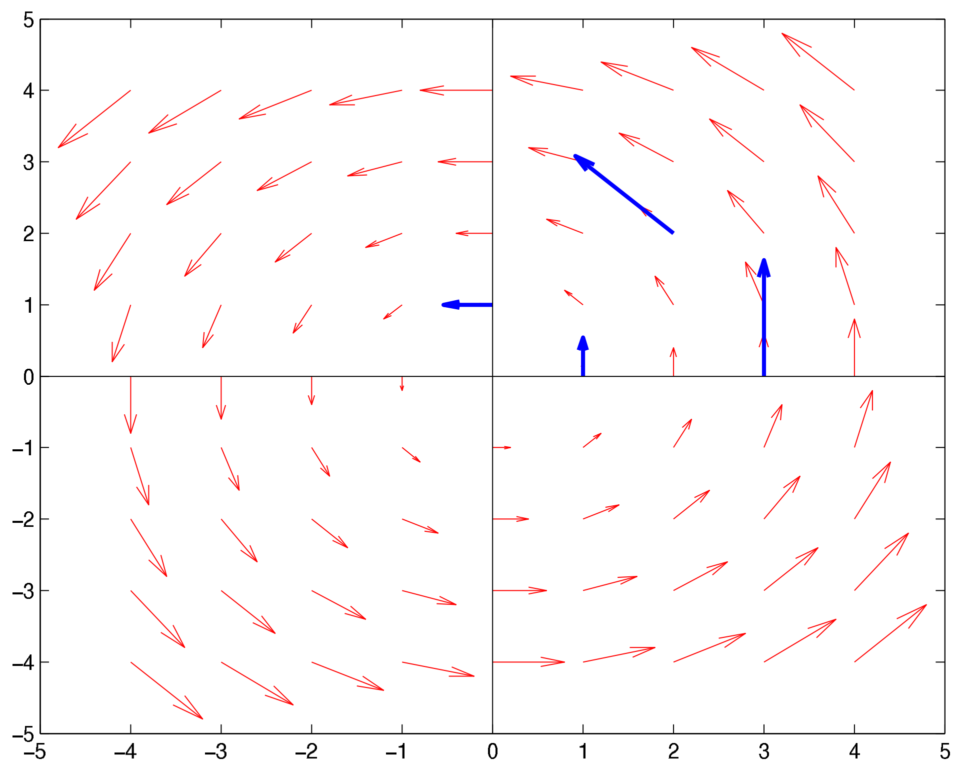


Vector field on $\mathbb{R}^2$

Definition: A vector field on $\mathbb{R}^2$ is a function $\mathbf{F}$ that assigns to each point (x, y) a two-dimensional vector $\mathbf{F}(x, y)$.

Example: $\mathbf{F}(x, y) = -y\mathbf{i} + x\mathbf{j}$

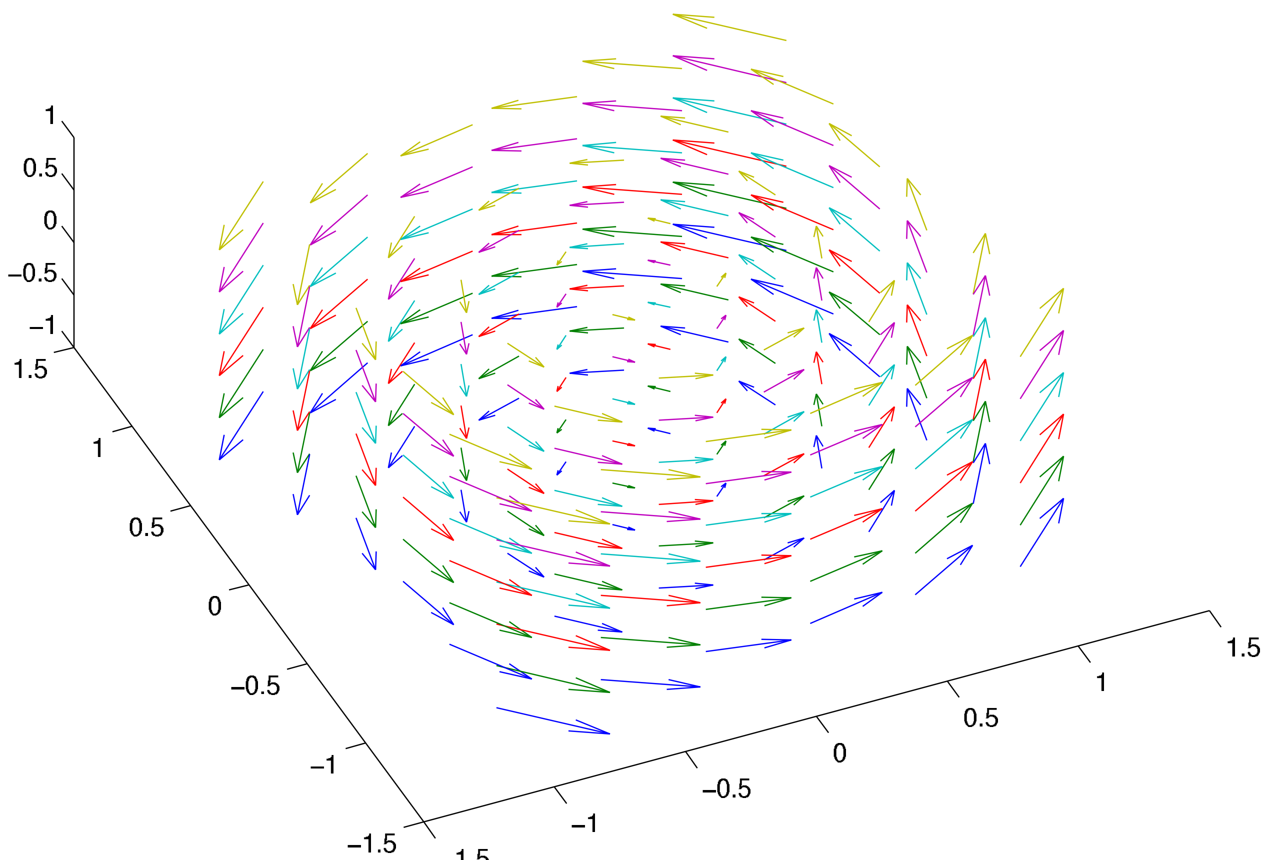
(x,y)	F(x,y)
(1,0)	<0,1>
(2,2)	<-2,2>
(3,0)	< 0,3>
(0,1)	<-1,0>
$\cdots$	$\cdots$



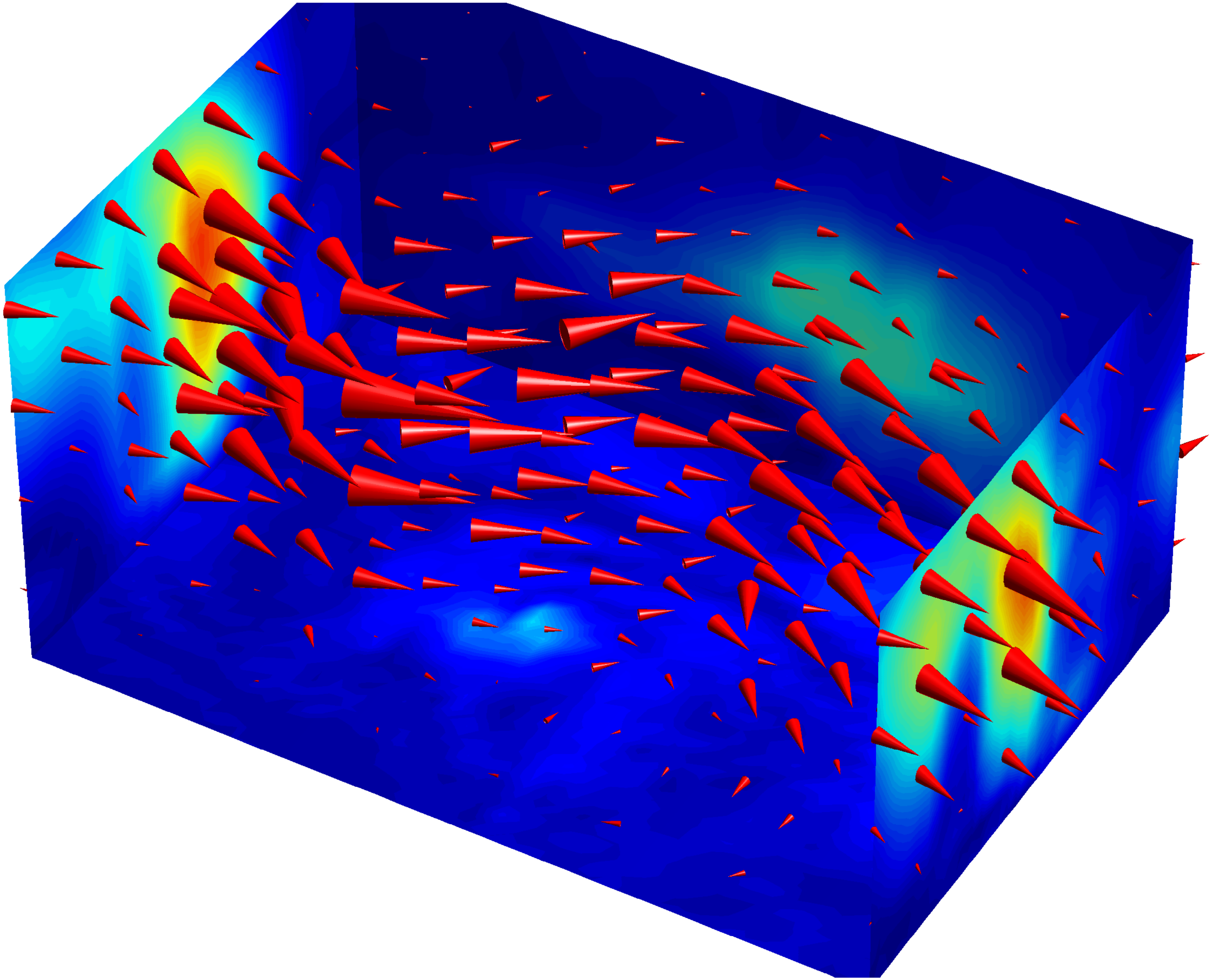
Vector field on $\mathbb{R}^3$

Definition: A vector field on $\mathbb{R}^3$ is a function $\mathbf{F}$ that assigns to each point (x, y, z) a three-dimensional vector $\mathbf{F}(x, y, z)$.

Example: $\mathbf{F}(x, y, z) = -y\mathbf{i} + x\mathbf{j} + 0\mathbf{k}$



Vector field



Gradient vector field

2D:

$$\nabla f(x, y) = f_x(x, y)\mathbf{i} + f_y(x, y)\mathbf{j}$$

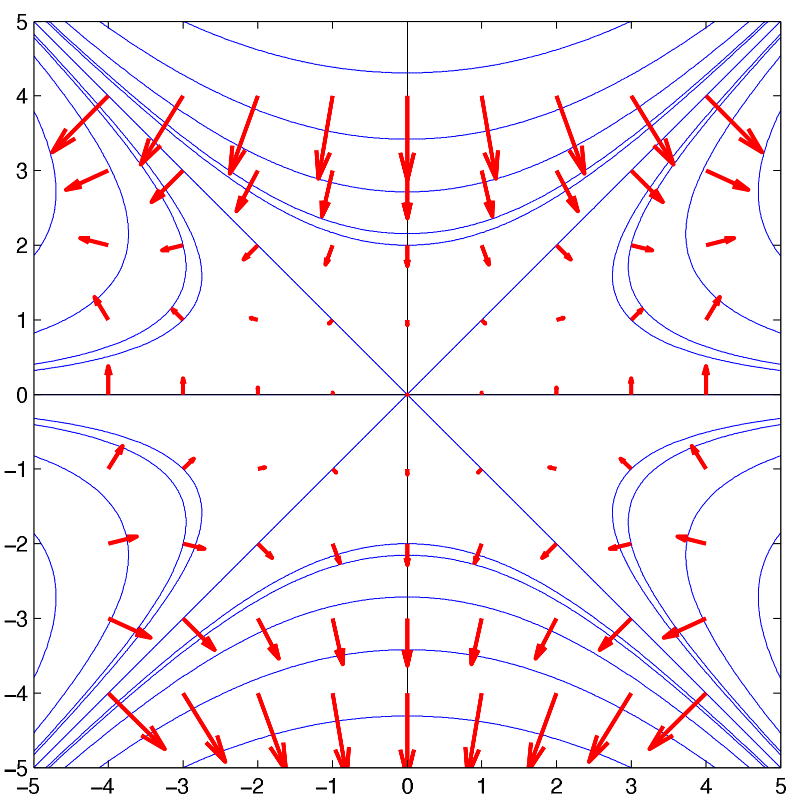
3D:

$$\nabla f(x, y, z) = f_x(x, y, z)\mathbf{i} + f_y(x, y, z)\mathbf{j} + f_z(x, y, z)\mathbf{k}$$

Gradient vector field

Eg. Find the gradient vector field of $f(x, y) = x^2y - y^3$.
Solution:

$$\nabla f(x, y) = f_x\mathbf{i} + f_y\mathbf{j} = 2xy\mathbf{i} + (x^2 - 3y^2)\mathbf{j}$$



Gradient vector field

Definition: A vector field $\mathbf{F}$ is called a **conservative vector field** if it is the gradient field of some scalar function, that is $\mathbf{F} = \nabla f$. In this case, f is called a **potential function** for $\mathbf{F}$.

Remark: Not all vector fields are conservative.