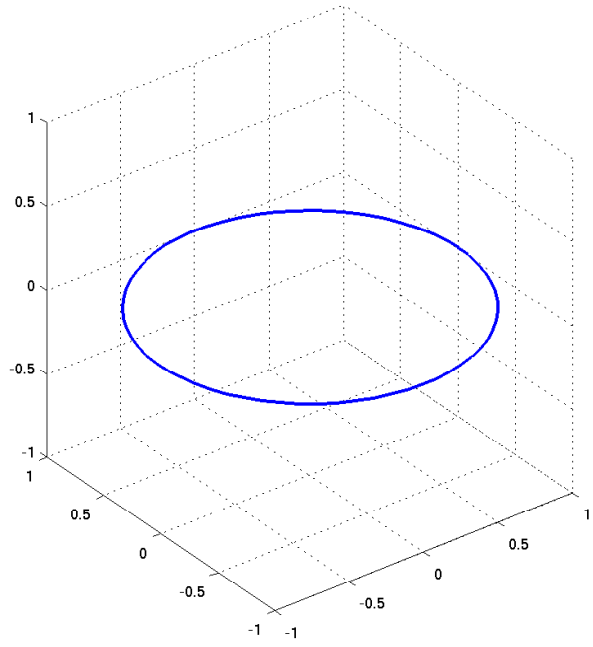
Cylinders and quadric surfaces

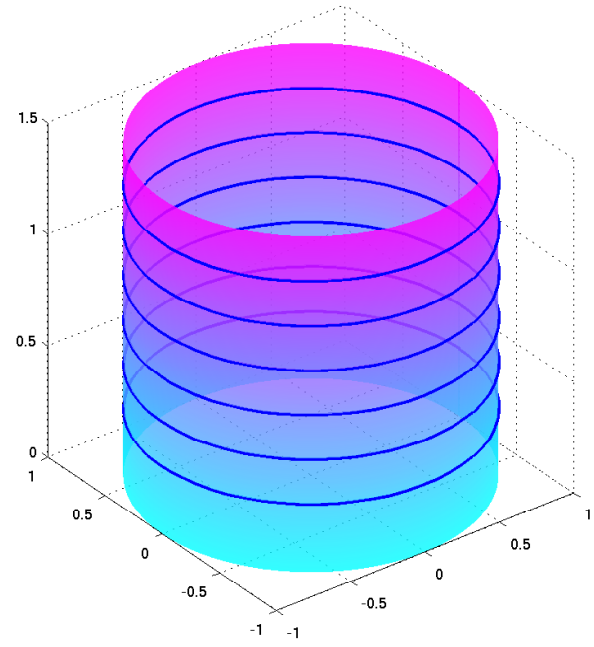
Math 2163

Cylinders

A cylinder is a surface that consists of all lines (rulings) that are parallel to a given line and pass through a given plane curve.

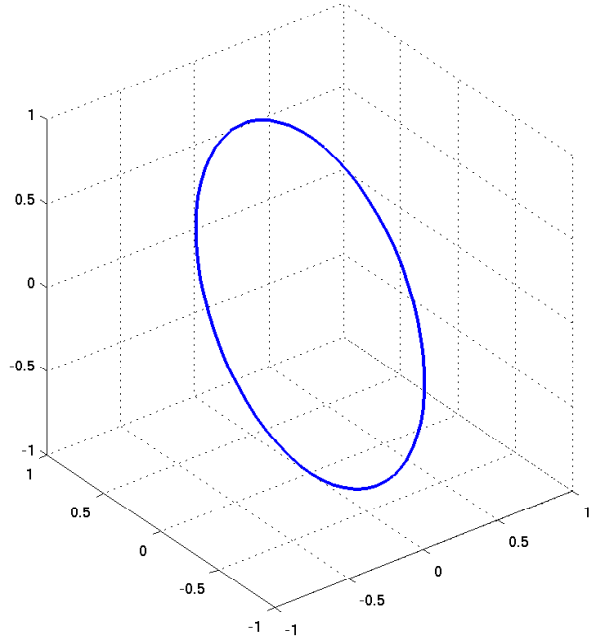
e.g. $x^2 + y^2 = 1$

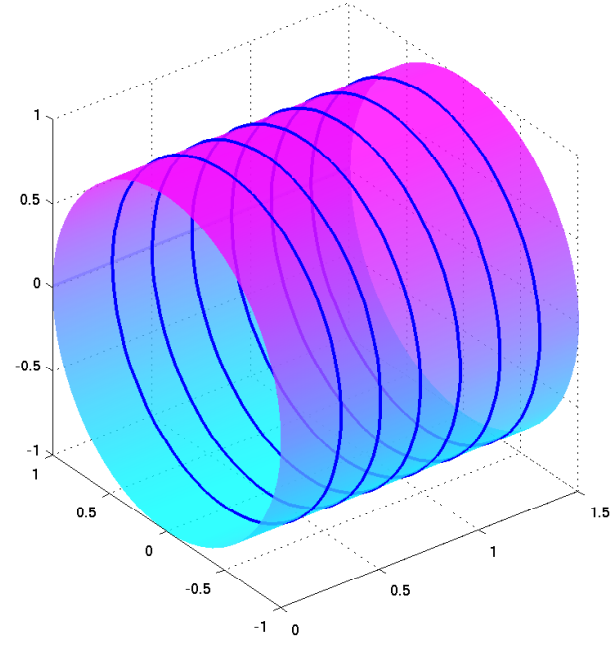




Cylinders

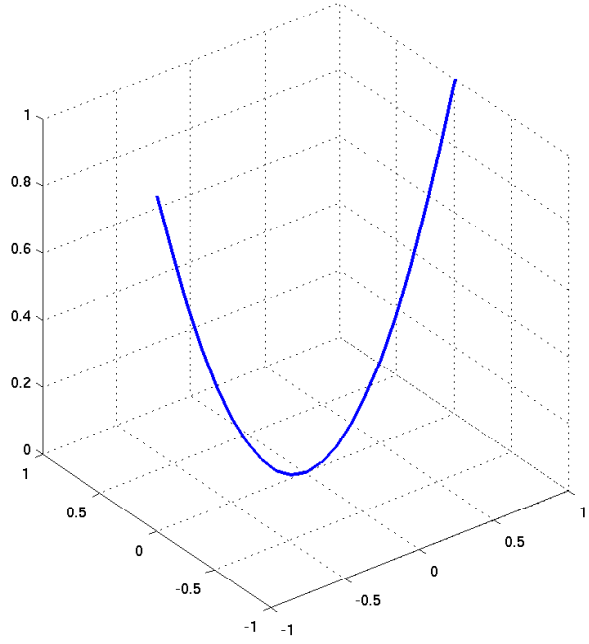
e.g. $y^2 + z^2 = 1$

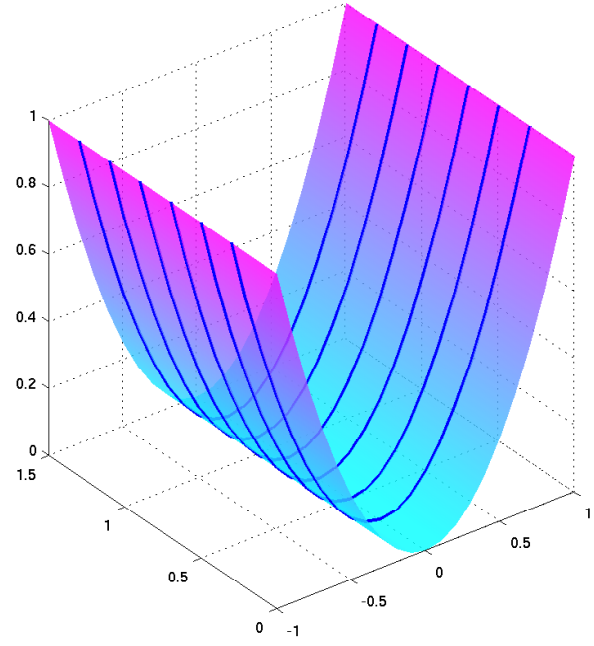




Cylinders

e.g. $z = x^2$





Quadric surfaces

A quadric surface is the graph of a second-degree equation in three variables x, y and z. e.g.

$$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + Gx + Hy + Iz + J = 0$$

$$Ax^2 + By^2 + Cz^2 + J = 0$$

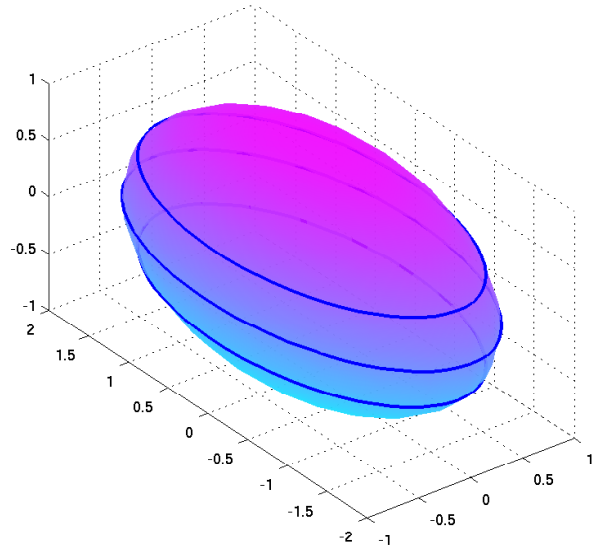
$$Ax^2 + By^2 + Iz = 0$$

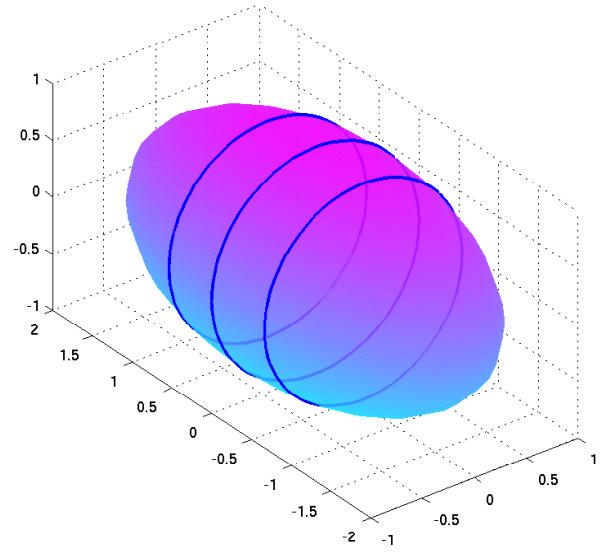
We will study the following types of quadric surfaces:

ellipsoid paraboloid hyperboloid cone

ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

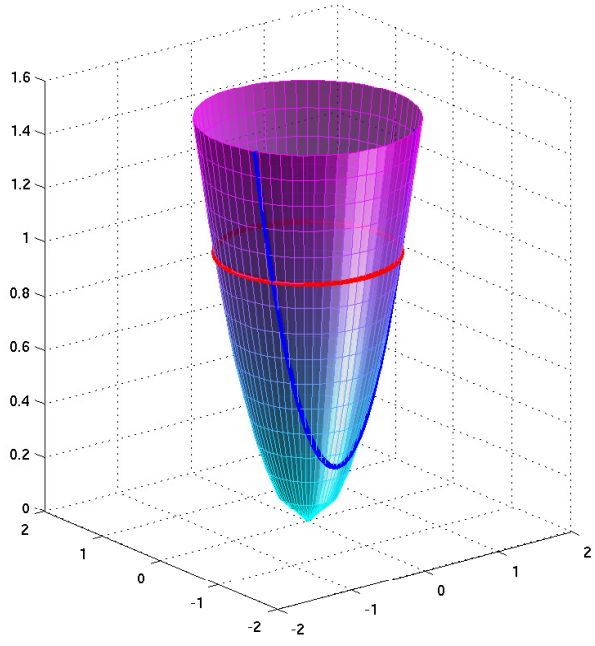


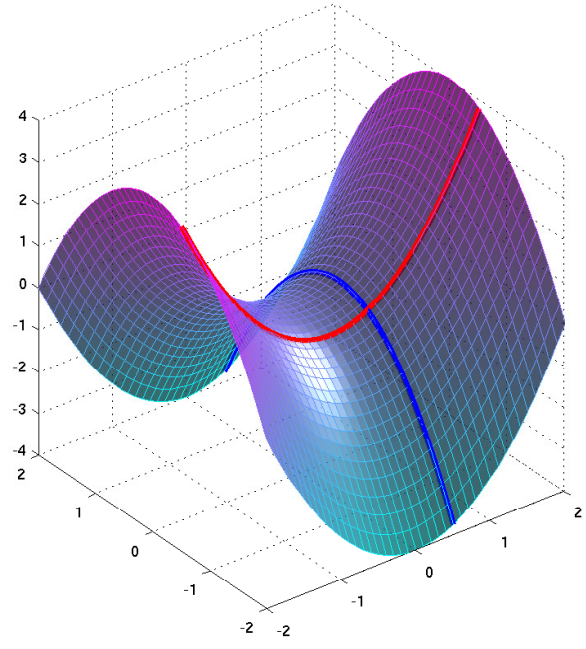


Paraboloid

Elliptic Paraboloid $$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

Hyperbolic Paraboloid $$\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$

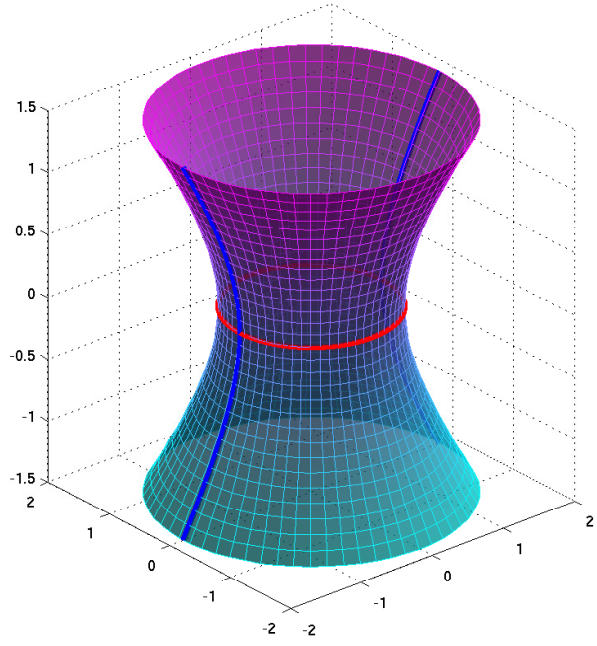


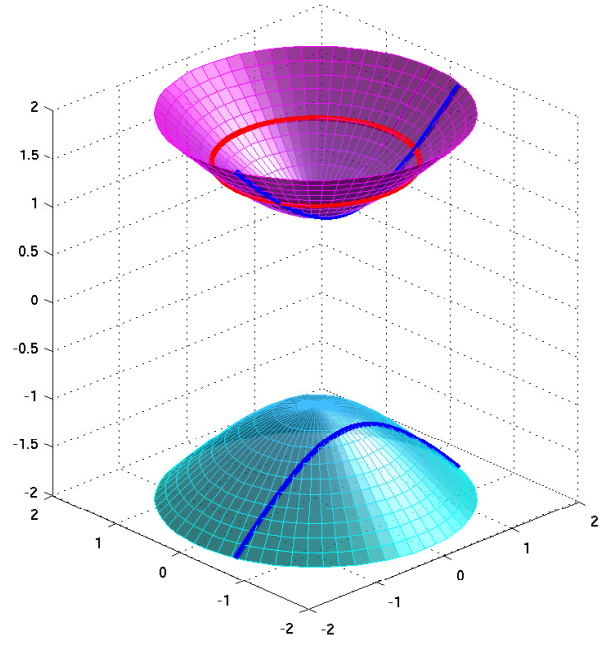


Hyperboloid

Hyperboloid of one sheet $$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

Hyperboloid of two sheets $$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$





Cone

Cone 1 $$\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

Cone 2 $$\frac{y^2}{b^2} = \frac{x^2}{a^2} + \frac{z^2}{c^2}$$