Practice Exam for midterm III

1. Find the radius of convergence and the interval of convergence of $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n+4}$.
(Solution: Radius of convergence is 1 and interval of convergence is $(-1, 1]$.)

2. Find a power series representation centered at 0 for $f(x) = \frac{x}{9+x^2}$.
(Solution: $f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{9^{n+1}}$

3. Find the first 5 terms in the Taylor series representation centered at $a = 1$ for $f(x) = \sqrt{x}$.
(Solution: $1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3 - \frac{5}{128}(x-1)^4 + \cdots$)

4. Use Taylor series to evaluate the integral $\int \frac{\sin x}{x}\, dx$.
(Solution: $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!(2n+1)} + C$)

5. Eliminate the parameter t to find a Cartesian equation of the curve $\begin{cases} x = 10\ln(9t) \\ y = \sqrt{t} \end{cases}$
(Solution: $y = \sqrt{\frac{e^{x/10}}{9}}$ or $x = 10\ln(9y^2)$)

6. Find an equation of the tangent line at the point corresponding to $t = 1$ for the curve $\begin{cases} x = e^{\sqrt{t}} \\ y = t - \ln(t^9) \end{cases}$ (Solution: $(y-1) = -\frac{16}{e}(x-e)$)

7. Find the points on the curve where the tangent is horizontal:
$$x = 13(\cos\theta - \cos^2\theta), \qquad y = 13(\sin\theta - \sin\theta\cos\theta)$$
(Solution: $(-39/4,\ -39\sqrt{3}/4)$, $(-39/4,\ 39\sqrt{3}/4)$)

8. Find the area of the surface obtained by rotating the curve about the x-axis
$$x = a\cos^3\theta, \qquad y = a\sin^3\theta, \qquad 0 \le \theta \le \pi$$
(Solution: $12\pi a^2/5$.)

9. Find the length of the curve $x = \frac{t}{1+t}$, $y = \ln(1+t)$, $0 \le t \le 2$.
(Solution: $-\sqrt{10}/3 + \ln(3+\sqrt{10}) + \sqrt{2} - \ln(1+\sqrt{2})$.)

10. Find the slope of the tangent line to the polar curve $r = 1/\theta$ at $\theta = \pi$.
(Solution: $-\pi$)

11. Find the area bounded by the curve $r = \sqrt{\sin\theta}$ and lies in the sector $0 \le \theta \le 2\pi/3$.
(Solution: $3/4$)

12. Find the length of the polar curve $r = 7\cos\theta$ for $0 \le \theta \le 3\pi/4$.
(Solution: $21\pi/4$)