Table of integrals

$$\int u\,dv = u\,v - \int v\,du \qquad \int a^u\,du = \frac{a^u}{\ln a} + C$$

$$\int \sec^2 u\,du = \tan u + C \qquad \int \csc^2 u\,du = -\cot u + C$$

$$\int \sec u \tan u\,du = \sec u + C \qquad \int \csc u \cot u\,du = -\csc u + C$$

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\frac{u}{a} + C \qquad \int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$$

Moments and centers of mass

$$M_y = \rho \int_a^b x[f(x) - g(x)]\,dx, \qquad M_x = \rho \int_a^b \frac{1}{2}[f^2(x) - g^2(x)]\,dx$$

$$\bar{x} = \frac{M_y}{\rho A}, \qquad \bar{y} = \frac{M_x}{\rho A}$$

Taylor series

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n \quad = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

Arc length and surface area

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx = \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy$$

$$\text{arc length } = \int ds, \qquad \text{surface area } = \begin{cases} \int 2\pi y\,ds & \text{rotate around x-axis} \\ \int 2\pi x\,ds & \text{rotate around y-axis} \end{cases}$$

Arc length and surface area (parametric equation)

$$\text{arc length } = \int_\alpha^\beta \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

$$\text{surface area } = \begin{cases} \int_\alpha^\beta 2\pi y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt & \text{rotate around x-axis} \\ \int_\alpha^\beta 2\pi x \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt & \text{rotate around y-axis} \end{cases}$$

Area and arc length (polar coordinates)

$$\text{area } = \int_\alpha^\beta \frac{1}{2} r^2\,d\theta$$

$$\text{arc length } = \int_\alpha^\beta \sqrt{r^2 + (\frac{dr}{d\theta})^2}\,d\theta$$