

Trigonometry

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\sin(-\theta) = -\sin \theta$$

$$\tan(-\theta) = -\tan \theta$$

$$\cos(\pi/2 - \theta) = \sin \theta$$

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\sin 2x = 2 \sin x \cos x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\sin(\pi/2 - \theta) = \cos \theta$$

$$\tan(\pi/2 - \theta) = \cot \theta$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

$$\begin{aligned} \cos 2x &= \cos^2 x - \sin^2 x \\ &= 2 \cos^2 x - 1 = 1 - 2 \sin^2 x \end{aligned}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\sin A \cos B = \frac{1}{2}[\sin(A - B) + \sin(A + B)]$$

$$\sin A \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$$

$$\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$$

Differentiation rules

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(\ln |x|) = \frac{1}{x}$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(a^x) = a^x \ln a$$

$$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$$

$$\frac{d}{dx}(\csc^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$$

Table of integrals

$$\int u dv = uv - \int v du$$

$$\int \frac{1}{u} du = \ln |u| + C$$

$$\int e^u du = e^u + C$$

$$\int a^u du = \frac{a^u}{\ln a} + C$$

$$\int \sin u du = -\cos u + C$$

$$\int \cos u du = \sin u + C$$

$$\int \sec^2 u du = \tan u + C$$

$$\int \csc^2 u du = -\cot u + C$$

$$\int \sec u \tan u du = \sec u + C$$

$$\int \csc u \cot u du = -\csc u + C$$

$$\int \tan u du = \ln |\sec u| + C$$

$$\int \cot u du = \ln |\sin u| + C$$

$$\int \sec u du = \ln |\sec u + \tan u| + C$$

$$\int \csc u du = \ln |\csc u - \cot u| + C$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1} \frac{u}{a} + C$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$$

$$\int \frac{du}{\sqrt{u^2+a^2}} = \ln(u + \sqrt{u^2+a^2}) + C$$

$$\int \frac{du}{\sqrt{u^2-a^2}} = \ln|u + \sqrt{u^2-a^2}| + C$$

Arc length and surface area

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

$$\text{arc length} = \int ds$$

$$\text{surface area} = \begin{cases} \int 2\pi y ds & \text{rotate around x-axis} \\ \int 2\pi x ds & \text{rotate around y-axis} \end{cases}$$