Trigonometry

$\csc\theta = \dfrac{1}{\sin\theta}$

$\sec\theta = \dfrac{1}{\cos\theta}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$\cot\theta = \dfrac{1}{\tan\theta}$

$\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

$\sin(\pi/2 - \theta) = \cos\theta$

$\cos(\pi/2 - \theta) = \sin\theta$

$\tan(\pi/2 - \theta) = \cot\theta$

$\sin(x+y) = \sin x\cos y + \cos x\sin y$

$\cos(x+y) = \cos x\cos y - \sin x\sin y$

$\sin(x-y) = \sin x\cos y - \cos x\sin y$

$\cos(x-y) = \cos x\cos y + \sin x\sin y$

$\tan(x+y) = \dfrac{\tan x + \tan y}{1 - \tan x\tan y}$

$\tan(x-y) = \dfrac{\tan x - \tan y}{1 + \tan x\tan y}$

$\sin 2x = 2\sin x\cos x$

$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$

$\sin^2 x = \dfrac{1 - \cos 2x}{2}$

$\cos^2 x = \dfrac{1 + \cos 2x}{2}$

$$\sin A\cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$$

$$\sin A\sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$$

$$\cos A\cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$$

Differentiation rules

$\dfrac{d}{dx}(\sin x) = \cos x$

$\dfrac{d}{dx}(\cos x) = -\sin x$

$\dfrac{d}{dx}(\tan x) = \sec^2 x$

$\dfrac{d}{dx}(\cot x) = -\csc^2 x$

$\dfrac{d}{dx}(\csc x) = -\csc x\cot x$

$\dfrac{d}{dx}(\sec x) = \sec x\tan x$

$\dfrac{d}{dx}(e^x) = e^x$

$\dfrac{d}{dx}(a^x) = a^x\ln a$

$\dfrac{d}{dx}(\ln|x|) = \dfrac{1}{x}$

$\dfrac{d}{dx}(\log_a x) = \dfrac{1}{x\ln a}$

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$$

$$\frac{d}{dx}(\csc^{-1} x) = -\frac{1}{x\sqrt{x^2-1}} \qquad \frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$$

Table of integrals

$$\int u\,dv = u\,v - \int v\,du \qquad \int \frac{1}{u}\,du = \ln|u| + C$$

$$\int e^u\,du = e^u + C \qquad \int a^u\,du = \frac{a^u}{\ln a} + C$$

$$\int \sin u\,du = -\cos u + C \qquad \int \cos u\,du = \sin u + C$$

$$\int \sec^2 u\,du = \tan u + C \qquad \int \csc^2 u\,du = -\cot u + C$$

$$\int \sec u \tan u\,du = \sec u + C \qquad \int \csc u \cot u\,du = -\csc u + C$$

$$\int \tan u\,du = \ln|\sec u| + C \qquad \int \cot u\,du = \ln|\sin u| + C$$

$$\int \sec u\,du = \ln|\sec u + \tan u| + C \qquad \int \csc u\,du = \ln|\csc u - \cot u| + C$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\frac{u}{a} + C \qquad \int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{u}{a} + C$$

$$\int \frac{du}{\sqrt{u^2+a^2}} = \ln(u + \sqrt{u^2+a^2}) + C \qquad \int \frac{du}{\sqrt{u^2-a^2}} = \ln|u + \sqrt{u^2-a^2}| + C$$

$$\int \frac{du}{a^2-u^2} = \frac{1}{2a}\ln\left|\frac{u+a}{u-a}\right| + C \qquad \int \frac{du}{u^2-a^2} = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + C$$

$$\int ue^{au}\,du = \frac{1}{a^2}(au-1)e^{au} + C \qquad \int \ln u\,du = u\ln u - u + C$$

Infinite sequences and series

$$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + \cdots = \begin{cases} \frac{a}{1-r} & \text{for } |r| < 1 \\ \text{divergent} & \text{otherwise} \end{cases}$$

$$S_n + \int_{n+1}^{\infty} f(x)\,dx \le S \le S_n + \int_n^{\infty} f(x)\,dx$$

Taylor series

$$\begin{aligned} f(x) &= \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n \\ &= f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots \end{aligned}$$

Some Maclaurin series and interval of convergence

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots \qquad (-1,\, 1)$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \qquad (-\infty,\, \infty)$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \qquad (-\infty,\, \infty)$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots \qquad (-\infty,\, \infty)$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots \qquad [-1,\, 1]$$

Area, arc length, and surface area

$$\begin{aligned} \text{area} &= \int_{\alpha}^{\beta} y \left(\frac{dx}{dt} \right) dt \\ \text{arc length} &= \int_{\alpha}^{\beta} \sqrt{\left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2}\, dt \\ \text{surface area} &= \begin{cases} \int_{\alpha}^{\beta} 2\pi y \sqrt{\left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2}\, dt & \text{rotate around x-axis} \\ \int_{\alpha}^{\beta} 2\pi x \sqrt{\left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2}\, dt & \text{rotate around y-axis} \end{cases} \end{aligned}$$

Area, arc length, in polar coordinates

$$\begin{aligned} \text{area} &= \int_{\alpha}^{\beta} \frac{1}{2} r^2\, d\theta \\ \text{arc length} &= \int_{\alpha}^{\beta} \sqrt{r^2 + (\frac{dr}{d\theta})^2}\, d\theta \end{aligned}$$