Quiz 6 – Math 2153, Calculus II – Oct. 7, 2011

1. Determine whether the series is convergent or divergent. If it is convergent, find its sum.
$$\sum_{n=1}^{\infty} \frac{1+2^n}{3^n}$$

Solution First, split up the general term into

$$\sum_{n=1}^{\infty} \frac{1+2^n}{3^n} = \sum_{n=1}^{\infty} \left(\frac{1}{3^n} + \frac{2^n}{3^n}\right)$$

Notice that

$$\sum_{n=1}^{\infty} \frac{1}{3^n} = \sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n = \frac{1}{3} + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 + \cdots$$
$$\sum_{n=1}^{\infty} \frac{2^n}{3^n} = \sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n = \frac{2}{3} + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \cdots$$

Both are geometric series. The first one with $r = 1/3$ and the second one with $r = 2/3$. Therefore both are convergent. By using the formula

$$a + ar + ar^2 + \cdots = \begin{cases} \frac{a}{1-r} & -1 < r < 1 \\ \text{diverges} & \text{otherwise} \end{cases}$$

we have

$$\sum_{n=1}^{\infty} \frac{1}{3^n} = \frac{1/3}{1-1/3} = \frac{1}{2}$$
$$\sum_{n=1}^{\infty} \frac{2^n}{3^n} = \frac{2/3}{1-2/3} = 2$$

Hence

$$\sum_{n=1}^{\infty} \frac{1+2^n}{3^n} = \sum_{n=1}^{\infty} \frac{1}{3^n} + \sum_{n=1}^{\infty} \frac{2^n}{3^n} = \frac{1}{2} + 2 = \frac{5}{2}$$

It converges to $\frac{5}{2}$.