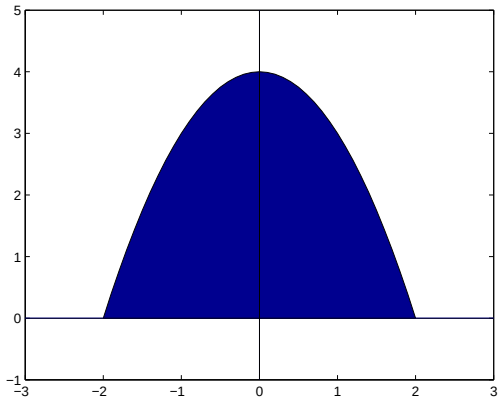
Quiz 5 – Math 2153, Calculus II – Sept. 30, 2011

1. Find the centroid of the region bounded by

$$y = 4 - x^2 \qquad \text{and} \qquad y = 0$$

Solution First, draw the graph of the region. Notice that $y = 4 - x^2$ and $y = 0$ intersects at $(-2,\, 0)$ and $(2,\, 0)$.



Let ρ be the density, then

$$m = \rho \int_{-2}^{2} (4 - x^2)\, dx = \frac{32}{3}\rho,$$

$$M_x = \rho \int_{-2}^{2} \frac{(4 - x^2)^2 - 0^2}{2}\, dx = \frac{256}{15}\rho,$$

$$M_y = \rho \int_{-2}^{2} x(4 - x^2)\, dx = 0$$

Hence the centroid is located at

$$\bar{x} = \frac{M_y}{m} = 0,$$

$$\bar{y} = \frac{M_x}{m} = \frac{8}{5}$$