1. Evaluate the integral

$$\int \frac{t-9}{t^2+3t-10}\,dt$$

**Solution** We use partial fractions to solve this problem. Notice that the denominator can be factored as

$$t^2+3t-10=(t+5)(t-2).$$

Therefore the integrand can be written as

$$\frac{t-9}{t^2+3t-10}=\frac{A_1}{t+5}+\frac{A_2}{t-2} \qquad (1)$$

Now we need to compute the values of $A_1$ and $A_2$. Notice that

$$\frac{A_1}{t+5}+\frac{A_2}{t-2}=\frac{A_1(t-2)}{(t+5)(t-2)}+\frac{A_2(t+5)}{(t+5)(t-2)}$$

$$=\frac{A_1(t-2)+A_2(t+5)}{(t+5)(t-2)}$$

So Equation (1) can be written as

$$\frac{t-9}{t^2+3t-10}=\frac{A_1(t-2)+A_2(t+5)}{(t+5)(t-2)}$$

Compare the numerators and denominators of the above equation, we have

$$t-9=A_1(t-2)+A_2(t+5) \qquad (2)$$

1. Set $t=2$ in Equation (2), we have

$$2-9=A_1(2-2)+A_2(2+5) \quad\Rightarrow\quad -7=0+7A_2 \quad\Rightarrow\quad A_2=-1$$

2. Set $t=-5$ in Equation (2), we have

$$-5-9=A_1(-5-2)+A_2(-5+5) \quad\Rightarrow\quad -14=-7A_1+0 \quad\Rightarrow\quad A_1=2$$

Plug in the values of $A_1$ and $A_2$ back into Equation (1), we have

$$\frac{t-9}{t^2+3t-10}=\frac{2}{t+5}-\frac{1}{t-2}$$

Hence the integral is

$$\int \frac{t-9}{t^2+3t-10}\,dt=\int\left(\frac{2}{t+5}-\frac{1}{t-2}\right)dt$$

$$=2\ln|t+5|-\ln|t-2|+C$$

(Or if you like, you can further simplify the answer into )

$$=\ln\left|\frac{(t+5)^2}{(t-2)}\right|+C$$