Quiz 2 – Math 2153, Calculus II – Sept. 2, 2011

1. Evaluate the integral

$$\int \frac{x}{\sqrt{x^2-7}}\,dx$$

Make sure your solution does NOT contain any inverse trigonometric function.

Solution 1 Use trigonometric substitution. Set $x = \sqrt{7}\sec\theta$, where $\theta \in [0, \frac{\pi}{2}) \cup [\pi, \frac{3\pi}{2})$, then $dx = \sqrt{7}\sec\theta\tan\theta\,d\theta$. The integral can be written as

$$\begin{aligned}
\int \frac{x}{\sqrt{x^2-7}}\,dx &= \int \frac{\sqrt{7}\sec\theta}{\sqrt{7\sec^2\theta-7}}\sqrt{7}\sec\theta\tan\theta\,d\theta \\
&= \int \frac{\sqrt{7}\sec\theta}{\sqrt{7}\tan\theta}\sqrt{7}\sec\theta\tan\theta\,d\theta \\
&= \int \sqrt{7}\sec^2\theta\,d\theta \\
&= \sqrt{7}\tan\theta + C
\end{aligned}$$

Now we need to substitute x back. Notice that $\sec\theta = x/\sqrt{7}$, we have

$$\tan\theta = \sqrt{\sec^2\theta - 1} = \sqrt{x^2/7-1}.$$

Hence the integral is

$$\int \frac{x}{\sqrt{x^2-7}}\,dx = \sqrt{7}\tan\theta + C = \sqrt{7}\sqrt{x^2/7-1} + C = \sqrt{x^2-7} + C$$

Solution 2 Use substitution. Set $u = x^2 - 7$, then $du = 2x\,dx$ The integral becomes

$$\int \frac{x}{\sqrt{x^2-7}}\,dx = \frac{1}{2\sqrt{u}}du = \sqrt{u} + C = \sqrt{x^2-7} + C$$