Quiz 10 – Math 2153, Calculus II – Nov. 18, 2011

1. Given the formula

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

   Use the series to evaluate the limit

$$\lim_{x\to 0} \frac{\sin x - x + \frac{1}{6}x^3}{x^5}$$

   Solution By using the series, we have

$$\begin{aligned}
\lim_{x\to 0} \frac{\sin x - x + \frac{1}{6}x^3}{x^5} &= \lim_{x\to 0} \frac{\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots\right) - x + \frac{1}{6}x^3}{x^5} \\
&= \lim_{x\to 0} \frac{\frac{x^5}{5!} - \frac{x^7}{7!} + \cdots}{x^5} \\
&= \lim_{x\to 0} \left(\frac{1}{5!} - \frac{x^2}{7!} + \cdots\right) \\
&= \frac{1}{5!} - 0 + \cdots
\end{aligned}$$

   where all the rest of terms are 0. Therefore

$$\lim_{x\to 0} \frac{\sin x - x + \frac{1}{6}x^3}{x^5} = \frac{1}{5!}$$