

Quiz 1 – Math 2153, Calculus II – Aug. 26, 2011

1. Evaluate the integral

$$\int_0^{\pi} t \sin 3t \, dt$$

Solution This integral can be computed using integration by parts. Set $f(t) = t$ and $g'(t) = \sin 3t$. Then we have $f'(t) = 1$, $g(t) = -\frac{1}{3} \cos 3t$ and

$$\begin{aligned} \int t \sin 3t \, dt &= t \left(-\frac{1}{3} \cos 3t \right) - \int 1 \times \left(-\frac{1}{3} \cos 3t \right) dt \\ &= -\frac{t}{3} \cos 3t + \frac{1}{3} \int \cos 3t \, dt \\ &= -\frac{t}{3} \cos 3t + \frac{1}{9} \sin 3t \end{aligned}$$

Therefore

$$\begin{aligned} \int_0^{\pi} t \sin 3t \, dt &= \left(-\frac{t}{3} \cos 3t + \frac{1}{9} \sin 3t \right) \Big|_{t=0}^{\pi} \\ &= \left(-\frac{\pi}{3} \cos 3\pi + \frac{1}{9} \sin 3\pi \right) - \left(0 + \frac{1}{9} \sin 0 \right) \\ &= \left(-\frac{\pi}{3}(-1) + 0 \right) - 0 \\ &= \frac{\pi}{3} \end{aligned}$$