Practice problems for midterm 3

1. Find the dimension of a rectangle with parimeter 100 m whose area is as large as possible.
2. Find a positive number such that the sum of this number and its reciprocal is as small as possible.
3. If 1200 cm^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box.
4. Find the point on the line $6x + y = 9$ that is closest to the point (-3,1).
5. Find the area of the largest rectangle that can be inscribed in the ellipse $x^2/a^2 + y^2/b^2 = 1$.
6. Find the most general antiderivative of the function
   (a) $f(x) = \frac{1}{2} + \frac{3}{4}x^2 - \frac{4}{5}x^3$
   (b) $f(x) = (x+1)(2x-1)$
   (c) $f(x) = 5x^{1/4} - 7x^{3/4}$
   (d) $f(x) = 6\sqrt{x} - \sqrt[6]{x}$
   (e) $f(u) = \frac{u^4 - 3\sqrt{u}}{u^2}$
   (f) $f(x) = 3e^x + 7\sec^2 x$
   (g) $f(t) = \sin t + 2\sinh t$
7. Find f
   (a) $f''(x) = 6x + 12x^2$
   (b) $f'(x) = 1 - 6x$ and $f(0) = 8$
8. Express the area under f as a limit.
   (a) $f(x) = \sqrt[4]{x}$, $1 \leq x \leq 16$, use R_n.
   (b) $f(x) = \frac{\ln x}{x}$, $3 \leq x \leq 10$, use L_n.
   (c) $f(x) = x\cos x$, $0 \leq x \leq \pi/2$, use M_n.
9. Determine the region whose area is equal to the given limit
   (a) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n}\left(5 + \frac{2i}{n}\right)^{10}$
   (b) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi}{4n} \tan\frac{i\pi}{4n}$
10. Find $\int_0^5 f(x)\,dx$ if
$$f(x) = \begin{cases} 3 & \text{for } x < 3 \\ x & \text{for } x \geq 3 \end{cases}$$
11. Use the properties of integrals to verify the inequality
$$2 \leq \int_{-1}^{1} \sqrt{1 + x^2}\,dx \leq 2\sqrt{2}$$

12. Find the derivative of the function

   (a) $F(x) = \int_x^\pi \sqrt{1 + \sec t}\, dt$

   (b) $h(x) = \int_2^{1/x} \arctan t\, dt$

   (c) $y = \int_{e^x}^0 \sin^3 t\, dt$

13. Evaluate the integral

   (a) $\int_{-1}^2 (x^3 - 2x)\, dx$

   (b) $\int_0^1 x^{4/5}\, dx$

   (c) $\int_1^9 \frac{x-1}{\sqrt{x}}\, dx$

   (d) $\int_0^{\pi/4} \sec^2 t\, dx$

   (e) $\int_{-1}^1 e^{u+1}\, du$

   (f) $\int (1-t)(2+t^2)\, dt$

   (g) $\int \sec t(\sec t + \tan t)\, dt$

   (h) $\int (4\sin\theta - 3\cos\theta)\, d\theta$

   (i) $\int_{-1}^2 (x - 2|x|)\, dx$

14. Use the substitution rule to compute

   (a) $\int x \sin x^2\, dx$

   (b) $\int (x+1)\sqrt{2x + x^2}\, dx$

   (c) $\int \frac{dx}{5-3x}$

   (d) $\int \sin(\pi t)\, dt$

   (e) $\int \frac{x}{x^2+1}\, dx$

   (f) $\int \frac{(\ln x)^2}{x}\, dx$

   (g) $\int e^x \sqrt{1 + e^x}\, dx$

   (h) $\int_0^7 \sqrt{4 + 3x}\, dx$

   (i) $\int_0^1 x^2(1 + 2x^3)^5\, dx$

   (j) $\int_1^2 \frac{e^{1/x}}{x^2}\, dx$

   (k) $\int_1^2 x\sqrt{x-1}\, dx$

15. Sketch the region between two curves and find its area.

   (a) $x = 1 - y^2$ and $x = y^2 - 1$

   (b) $y = \sin(\pi x/2)$ and $y = x$

16. Find the volume of the solid obtained by rotating the curve along specific axis

   (a) $y = x^3$, $y = x$, $x \geq 0$, about the $x-$ axis

   (b) $y^2 = x$, $x = 2y$, about the $y-$ axis