Practice problems for midterm 2

1. Find the first and second derivative of
   (a) $f(x) = \left(\sqrt{x} + \frac{1}{\sqrt[3]{x}}\right)^2$
   (b) $f(x) = \frac{x+1}{x^3+x-2}$
   (c) $f(x) = \frac{\sin x}{\cos x+7}$
   (d) $f(x) = x^7 \sin x \tan x$
   (e) $f(x) = \sqrt{x + \sqrt{x}}$
   (f) $f(x) = \sqrt[3]{1 + \tan x}$
   (g) $f(x) = e^{x \cos x}$
   (h) $f(x) = \sin(\tan x)$
   (i) $f(x) = \sin^{-1}(2x + 1)$
   (j) $f(x) = \ln(x\sqrt{x-1})$
   (k) $f(x) = \cosh(\ln x)$
   (l) $f(x) = x \sinh x - \cosh x$

2. Use implicit differentiation to compute $\frac{dy}{dx}$.
   (a) $x^3 + y^3 = 1$
   (b) $\sqrt{x+y} = 1 + xry^2$
   (c) $4 \cos x \sin y = 1$
   (d) $1 + x = \sin(xy^2)$
   (e) $e^y \cos x = 1 + \sin(xy)$

3. Find the limit of
   (a) $\lim_{x\to 0} \frac{\sin 4x}{\sin 6x}$
   (b) $\lim_{t\to 0} \frac{\sin^2 3t}{t^2}$
   (c) $\lim_{\theta\to 0} \frac{\cos\theta - 1}{\sin\theta}$

4. Use logarithmic differentiation to find $\frac{dy}{dx}$.
   (a) $y = (2x + 1)^5(x^4 - 3)^6$
   (b) $y = \sqrt[4]{\frac{x^2+1}{x^2-1}}$
   (c) $y = (\sin x)^{\ln x}$
   (d) $y = (\tan x)^{1/x}$

5. A bacteria culture grows at a rate proportional to its size. After 2 hours there are 600 bacteria and after 8 hours the count is $75,000$.
   (a) Find the initial population.
   (b) Find the expression for the population after t hours.

(c) Find the number of cell after 5 hours.

(d) Find the growth rate after 5 hours.

6. Each side of a square is increasing at a rate of 6 cm/s. At what rate is the area of the square increasing when the area of the square is 16 cm^2.

7. A cylindrical tank with radius 5 m is being filled with water at a rate of 3 m^3/min. How fast is the height of the water increasing?

8. A street light is mounted at the top of a 15-ft-tall pole. A man 6 ft tall walks away from the pole with a speed of 5 ft/s along a straight path. How fast is the tip of his shadow moving when he is 40 ft from the pole?

9. At noon, ship A is 150 km west of ship B. Ship A is sailing east at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4:00 pm?

10. Find the linear approximation of the function $f(x) = e^x$ at $a = 0$.

11. Use the linear approximation of $y = (2 + x)^5$ at $a = 0$ to estimate the value of $(2.001)^5$.

12. Use the linear approximation of $y = e^x$ at $a = 0$ to estimate the value of $e^{-0.015}$.

13. Find the differential dy of each function

    (a) $y = \ln \sqrt{1 + t^2}$

    (b) $y = \frac{u+1}{u-1}$

14. If $\tanh x = \frac{4}{5}$, find the values of $\sinh x$ and $\cosh x$.

15. Find the critical numbers of the function.

    (a) $f(x) = x^3 + x^2 - x$

    (b) $f(x) = x^{4/5}(x - 4)^2$

    (c) $f(x) = x^2 e^{-3x}$

16. Find the absolute maximum and absolute minimum of the function on the given interval.

    (a) $f(x) = 3x^2 - 12x + 5$ on $[0, 3]$

    (b) $f(x) = \frac{x}{x^2+1}$ on $[0, 2]$

    (c) $f(x) = x - \ln x$ on $[0.5, 2]$

    (d) $f(x) = xe^{-x^2/8}$ on $[-1, 4]$

17. For the given function $f(x)$, (a) find the intervals where f is increasing or decreasing; (b) find the local maximum and minimum values of f using both the first derivative test and the second derivative test; (c) find the intervals of concavity and the inflection points.

    (a) $f(x) = 2x^3 + 3x^2 - 36x$

    (b) $f(x) = \frac{x^2}{x^2+3}$

    (c) $f(x) = x^2 \ln x$

18. Suppose that $5 \leq f'(x) \leq 10$ for all values of x, then $\underline{\ ?\ } \leq f(4) - f(-1) \leq \underline{\ ?\ }$

19. Find the limit

    (a) $\lim_{x\to 1} \frac{x^2-1}{x^2-x}$

    (b) $\lim_{t\to 0} \frac{e^{3t}-1}{t}$

    (c) $\lim_{x\to 0} \frac{x-\sin x}{x-\tan x}$

    (d) $\lim_{x\to \infty} \frac{\ln x}{\sqrt{x}}$

    (e) $\lim_{x\to \infty} \frac{x+x^2}{1-2x^2}$

20. Find an equation of the slant asymptote.

    (a) $y = \frac{x^2+1}{x+1}$

    (b) $y = \frac{5x^4+x^2+x}{x^3-x^2+2}$