Practice problems for midterm 1

1. Find an expression for the function $y = f(x)$ whose graph is the bottom half of the parabola $x + (8 - y)^2 = 0$.

2. A rectangle has parameter 24 m. Express the area of the rectangle as a function $A(l)$ of the length l of one side.

3. The monthly cost of driving a car depends on the number of miles driven. In December, it cost \$280 to drive 300 mi and in May it costs 420 to drive $1,000$ mi. Express the monthly cost C as a function of the distance driven d, assuming that a linear relationship gives a suitable model.

4. Find the domain of the functions

$$g(u) = \sqrt{u} - \sqrt{7 - u}$$
$$p(t) = \frac{t^2 + 1}{t^2 - 5t + 6}$$
$$h(s) = \frac{\ln(s + 1) + e^{2s}}{s^2 - 4s}$$
$$q(r) = \frac{1}{sin^{-1}r}$$

5. Find the range of the function $h(x) = \sqrt{25 - x^2}$.

6. If $f(x) = x + 6$ and $h(x) = 2x - 3$, find a function g such that $g \circ f = h$.

7. Sketch the graph of $y = 2\sin(2x + \pi) - 1$.

8. Find the inverse function of

$$f(x) = \frac{e^x - 10}{e^x + 1}$$
$$g(x) = \ln \frac{x - 1}{x}$$

9. Express the following function in a single logarithm:

$$f(x) = 2\log_5(x^2 + 1) - \log_5(\tan x) + \log_5 \sqrt{x + 1}$$

10. Solve the equation for x

$$e^{3x+4} = 10$$
$$\ln(x - 2) = 10$$

11. Find the exact value of

$$\tan(\sec^{-1}(6))$$
$$\sin(\cos^{-1}(\frac{1}{2}))$$
$$\sin(2\cos^{-1}(\frac{2}{3}))$$
$$\cos(2\sin^{-1}(\frac{2}{3}))$$

12. Determine the infinite limit

$$\lim_{x\to 0} \frac{x-1}{x^2(x+5)}$$
$$\lim_{x\to 4^+} \frac{(x^2+2)(x-1)}{x(x-4)}$$
$$\lim_{x\to 9^-} \ln(9-x^2)$$

13. Let $f(x) = x^3 - 2x^2 + 1$, compute $\lim_{h\to 0} \frac{f(a+h)-f(h)}{h}$.

14. Evaluate the limit (if it does not exist, write DNE)

$$\lim_{x\to 0} \frac{(2+x)^{-1} - 2^{-1}}{x}$$
$$\lim_{t\to 25} \frac{t-25}{5-\sqrt{t}}$$
$$\lim_{x\to 5} \frac{\sqrt{x+6}-1}{5x-x^2}$$
$$\lim_{x\to -3} \frac{x^2+3x}{|2x+6|}$$
$$\lim_{x\to \pi} \frac{\sin x}{2+\cos x}$$
$$\lim_{x\to 1} \arcsin \frac{1-\sqrt{x}}{1-x}$$

15. If $1 \le f(x) \le x^2 + 2x + 2$ for all x, find the limit $\lim_{x\to -1} f(x)$.

16. Evaluate the limit

$$\lim_{x\to 0} x^6 \cos \frac{2}{x}$$

17. Prove that the equation $x^3 - 6x + 1 = 0$ has a root in $[0, 1]$.

18. Evaluate the limits:

$$\lim_{x\to \infty} \frac{\sqrt{2x^2+1}}{3x-5}$$
$$\lim_{x\to -\infty} \frac{1-x-x^2}{2x^2-7}$$
$$\lim_{x\to \infty} \left(\sqrt{9x^2+x} - 3x\right)$$
$$\lim_{x\to -\infty} \left(x + \sqrt{x^2+2x}\right)$$
$$\lim_{x\to \infty} e^{-2x} \cos x$$
$$\lim_{x\to \infty} \tan^{-1}(x^2 - x^4)$$

19. Find the horizontal and vertical asymptotes of $y = \frac{x^2+1}{2x^2-3x-2}$.

20. Find the horizontal and vertical asymptotes of $y = \frac{2x^2+x-1}{x^2+x-2}$.

21. Sketch the graph of $y = (3-x)(1+x)^2(1-x)^4$.

22. Find an equation of the tangent line for $y = \frac{x-1}{x-2}$ at point (3, 2).

23. Find $f'(a)$

$$f(x) = 3 - 2x + 4x^2$$
$$f(x) = \frac{x^2+1}{x-2}$$
$$f(x) = \sqrt{3x+1}$$
$$f(x) = 1/\sqrt{3x+1}$$

24. If a ball is thrown into the air with a velocity of 40 fs/s, its height (in feet) after t seconds is given by $y = 40t - 16t^2$. Find the velocity when $t = 2$.

25. Understand the relation of graphs for $f(x)$ and $f'(x)$. Textbook, section 2.8, exercises 3, 35-38, 41-44.

26. Compute f', f'', f''' for

$$f(x) = 1 + 4x - x^2$$
$$f(x) = 1/x$$