Math 6490: Assignment 2  
Due Thursday, February 26

1. Penrose wallpaper:
   (a) Make lots of copies of the attached Penrose tiling. The tiles are all of two shapes: a skinny rhombus, with angles $36^\circ$ and $144^\circ$ and a fat rhombus with angles $72^\circ$ and $108^\circ$.
   (b) By trial and error, connect a subset of the vertices together with red line segments so that we can see a larger Penrose tiling superimposed on top of the smaller one.
   (c) Continue this pattern throughout the whole page.
   (d) Find the ratio of the sides of the larger tiles to the sides of the original tiles.
   (e) Show that each larger tile is subdivided in exactly the same way. Try to find rules for this subdivision.
   (f) Find a Euclidean similarity that maps the tiling by small tiles onto the tiling by large tiles.

2. Read Chapter 2 on complex number geometry.

3. Project 2.1: Find all the Gaussian primes $m + ni$ with $\sqrt{m^2 + n^2} \leq 20$. Make a plot. We will work on this in Maple.

   For extra credit, devise a program to factor a Gaussian integer $m + ni$ into prime Gaussian integers.

4. Project 2.2: If $x + iy = (u + iv)^2$, write out formulas for $u$ and $v$ in terms of $x$ and $y$. Explain how to choose the solution with $u > 0$ or $u = 0$ and $v \geq 0$.

5. Project 2.3: We will devise Maple programs to produce these kinds of pictures. Read the Maple help page on conformal.

6. Project 2.4: Explain how every rigid motion can be written in the form $T(z) = Az + B$ or $T(z) = A\overline{z} + B$, where $z$ is a complex variable and $A, B$ are complex constants. What about Euclidean similarities?

7. Project 2.5: We will work on drawing complex spirals in Maple.

8. Project 2.6: Prepare a group law table with $T_1, ..., T_8$ running along both the top of the columns and the left side of the rows. Then in the entry corresponding to the row labelled $T_i$ and the column labelled $T_j$ place the result of composition $T_i \circ T_j$. (It will be some $T_k$.) This will show the structure of the group of these 8 transformations.