### Project on Hex

**HEX** is an amazingly simple and yet difficult game invented by the Danish mathematician and poet Piet Hein in 1942, and independently by John Nash in 1948 as a graduate student in the math department at Princeton University, where it was called simply *Nash* or sometimes *John* partly to indicate that it could be readily played on hexagonal tiles commonly found in bathrooms.

The amazing story of John Nash, whose 27 page doctoral thesis in 1950 on game theory became a great classic of modern economics and for which he won the Nobel Prize in 1994, is told in one of the most compelling scientific biographies ever written, *A Beautiful Mind*, by Sylvia Nasar.

Piet Hein also became famous for many unusual discoveries, including the “Super Egg,” which used to very popular at science museum giftshops, as well as a cryptic style of poetry called “Grooks.” Some of Piet Hein’s grooks are on the attached pages, which I have borrowed from [www.mazeworks.com](http://www.mazeworks.com).

The rules of Hex are explained on the attached pages.

#### First project

Learn the rules of Hex and solve the three Hex problems invented by Piet Hein on the attached page. To solve this problem, you must give the move that wins the game for White, and you must explain how White can win no matter what Black plays after that. This requires a little bit of case-by-case analysis.

For some notation describing moves, you can label the spaces along White’s sides as *a*, *b*, *c*, etc., from left to right, and the spaces along Black’s sides as 1, 2, 3, etc. Then when you give a move as *d3*, for example, you are giving the coordinates of the space you place the stone in relation to the two players’ sides.

#### Second project

These are questions about the general *n* × *n* game of Hex.

1. First prove convincingly that there are no ties: one of the two players must win.
2. Secondly, prove that the player who goes first can always win.

John Nash discovered a very simple proof that the first player can always win, but it is entirely nonconstructive, meaning that he did not find a description of the strategy that works. He simply proved that there is one.

As far as I know, the largest size board for which an explicit winning strategy is known (and programmed!) is 7 × 7. You can play against the computer at [mazeworks.com](http://mazeworks.com) on a 7 × 7 board, but it will be a little frustrating. It seems the computer always goes first.