Fractals and how to make a Sierpinski Tetrahedron

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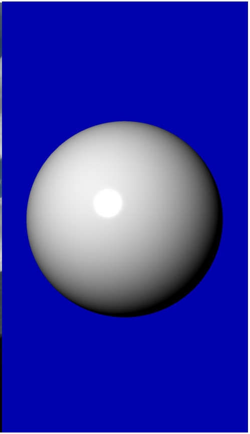
"Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line." - Benoît Mandelbrot

Images: Wikipedia



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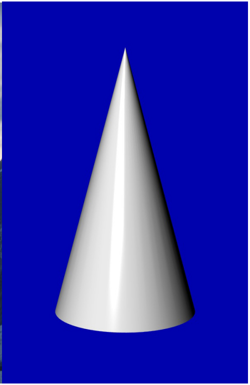

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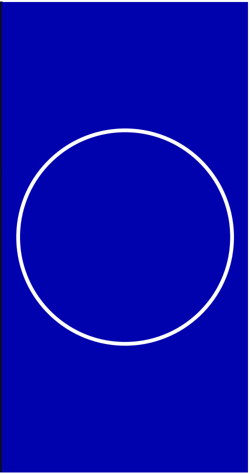

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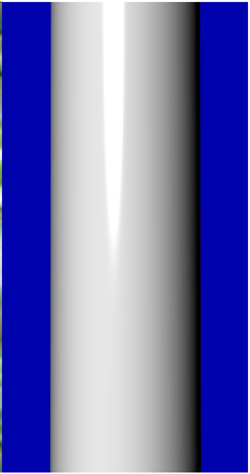

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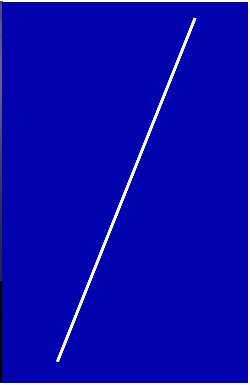

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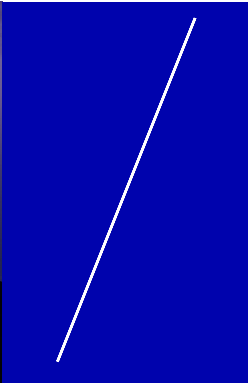

What makes these mathematical shapes bad approximations to these natural phenomena?

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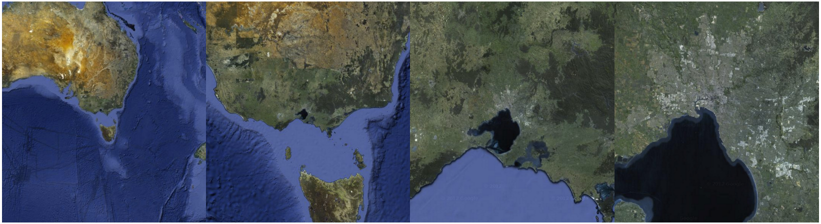
- One way is to say that something is rough if it has features at many different scales.



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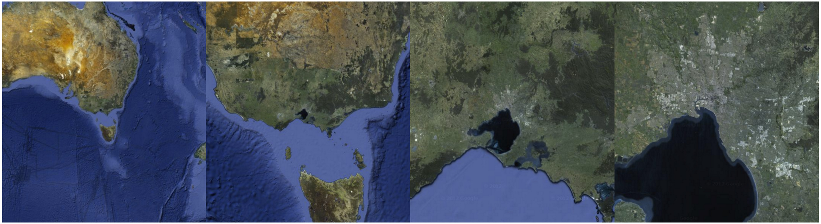

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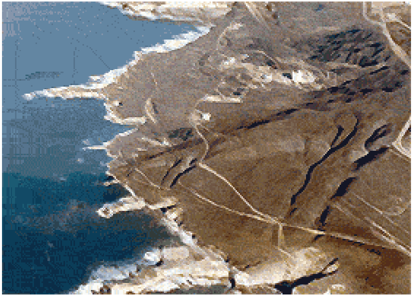
Are there "simple" rough things? Simple enough for us to try to look at using mathematics?

Self-similarity

An object is self-similar if it is similar to a part of itself.

That is, a small part of the object is the same as a larger part, scaled down.

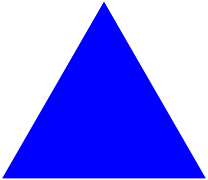


Barnsley's fern

Images: Wikipedia

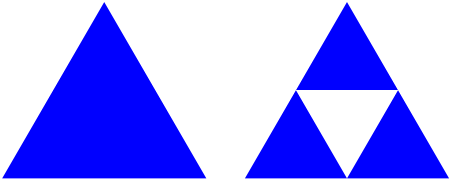
The Sierpinski Triangle

1. Start with a triangle.



The Sierpinski Triangle

1. Start with a triangle.
2. Cut it into four triangles and remove the middle one.



The Sierpinski Triangle

1. Start with a triangle.
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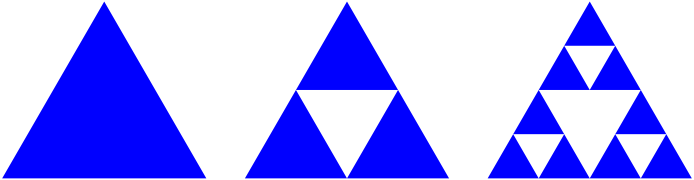
3. Repeat for the three new triangles.



The Sierpinski Triangle

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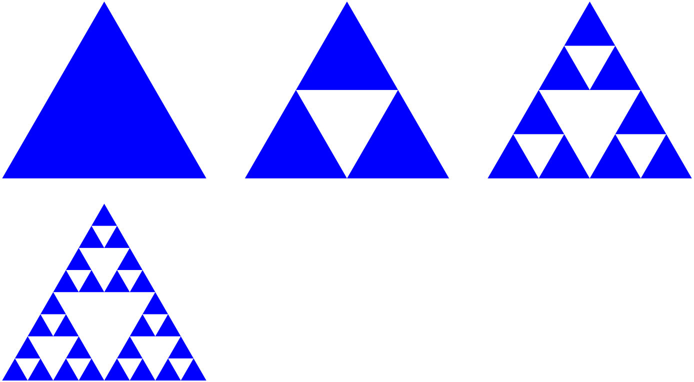
3. Repeat for the three new triangles.
4. Keep going forever.



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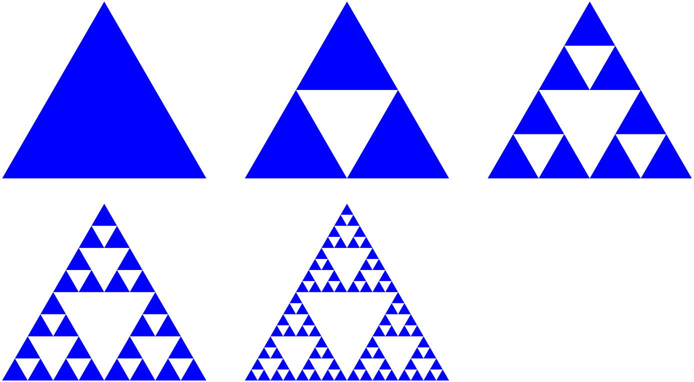

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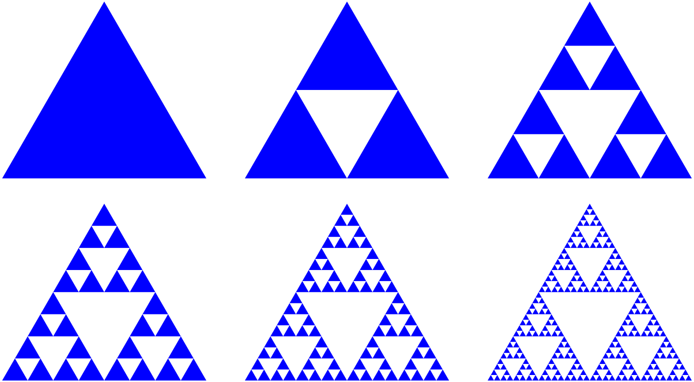
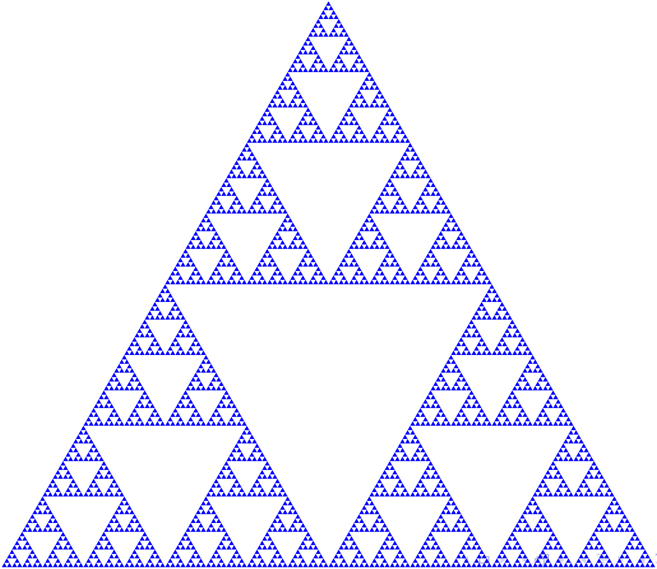
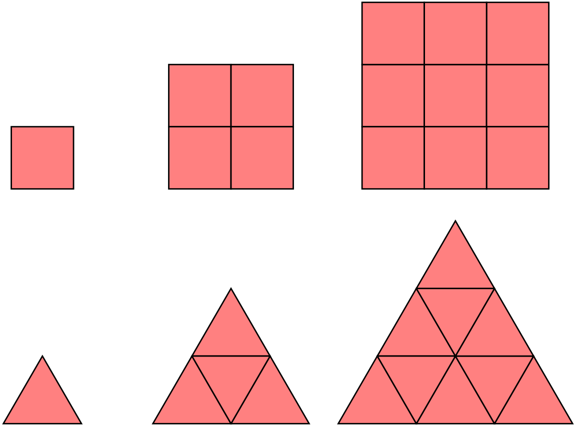


The Sierpinski Triangle

The Sierpinski triangle is self-similar because it is made up of 3 smaller copies of itself, each half the size of the original.

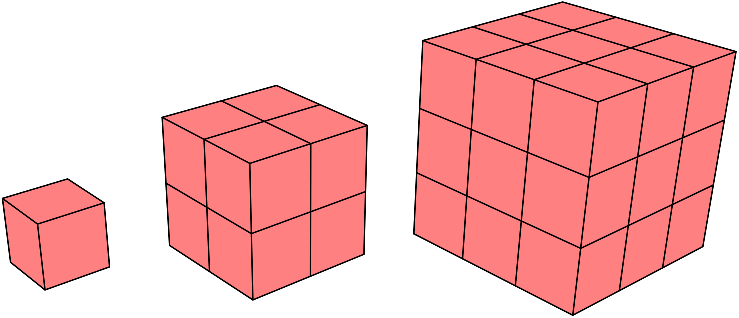


Dimension



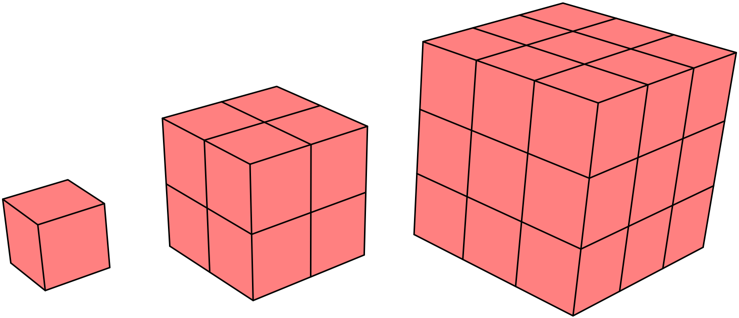
If you double the size of a square or triangle, you can make it from 4 copies of the original. If you triple the size, you can make it from 9 copies of the original.

Dimension



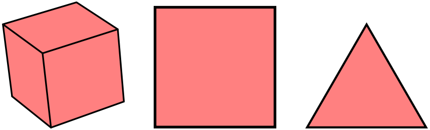
If you double the size of a cube...?
If you triple the size...?

Dimension



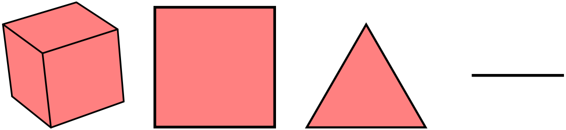
If you double the size of a cube, you can make it from 8 copies of the original.
If you triple the size, you can make it from 27 copies of the original.

Dimension



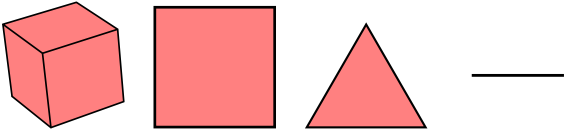
	$\times 1$	$\times 2$	$\times 3$	
Cube	1	8	27	
Square	1	4	9	
Triangle	1	4	9	

Dimension



	$\times 1$	$\times 2$	$\times 3$	
Cube	1	8	27	
Square	1	4	9	
Triangle	1	4	9	
Line segment				

Dimension



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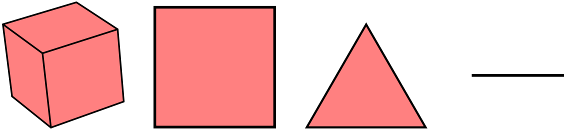
Line segment	1	2	3	

Dimension



	$\times 1$	$\times 2$	$\times 3$	Dimension
Cube	1	$8 = 2^3$	$27 = 3^3$	
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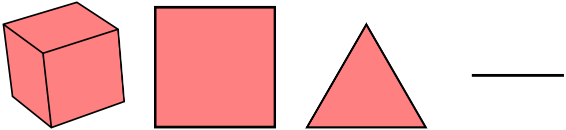
Triangle	1	$4 = 2^2$	$9 = 3^2$	
Line segment	1	$2 = 2^1$	$3 = 3^1$	

Dimension



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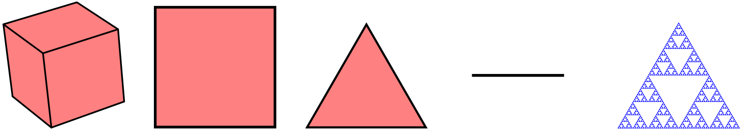
Triangle	1	$4 = 2^{2}$	$9 = 3^{2}$	2
Line segment	1	$2 = 2^{1}$	$3 = 3^{1}$	1

Dimension



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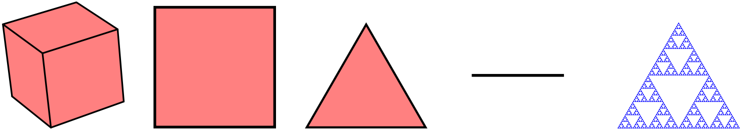
Line segment	1	$2 = 2^1$	$3 = 3^1$	1
Sierpinski triangle	1	3		

Dimension



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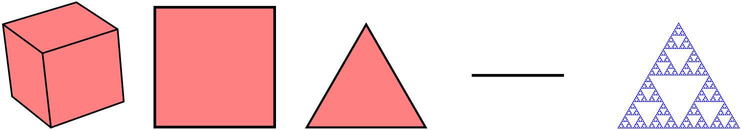
Sierpinski triangle	1	$3 = 2^?$		?

Dimension



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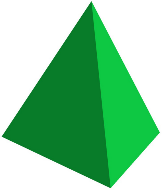
Line segment	1	$2 = 2^1$	$3 = 3^1$	1
Sierpinski triangle	1	$3 = 2^?$		?

The equation $3 = 2^x$ can be solved by $x = \frac{\log(3)}{\log(2)} \approx 1.585$, so this suggests that the Sierpinski triangle has a "fractional" dimension, between 1 and 2!

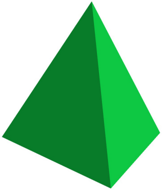
The Sierpinski Tetrahedron

1. Start with a tetrahedron.



The Sierpinski Tetrahedron

1. Start with a tetrahedron.
2. Remove the middle to leave four tetrahedra.



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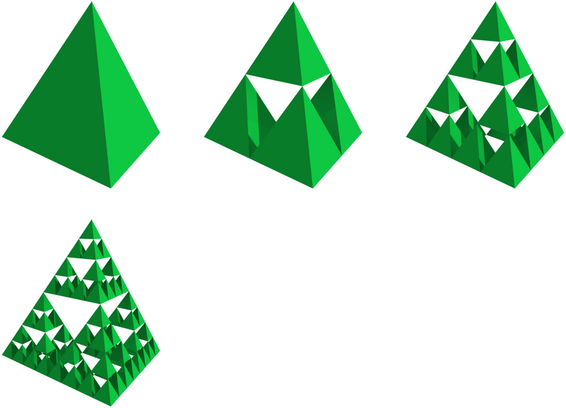

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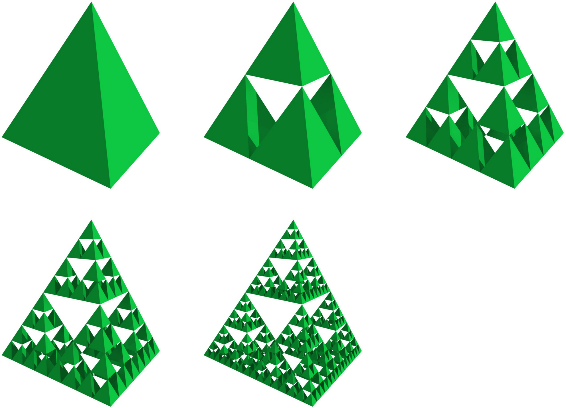

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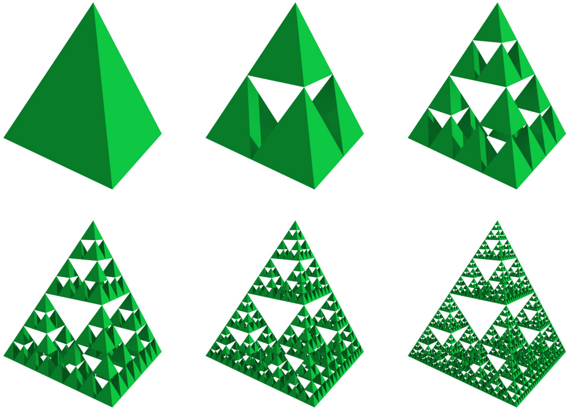
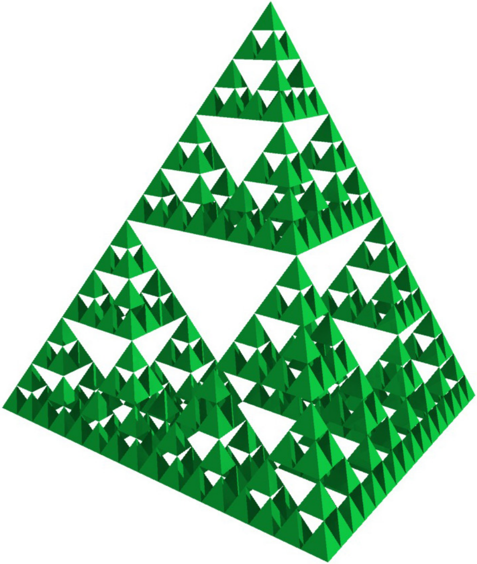


The Sierpinski Tetrahedron: Activity

We will be attempting to build this one out of small tetrahedra:



Questions:

- How many tetrahedra will we need?
- What is the shape that gets removed from the middle of each tetrahedron?
- What is the "dimension" of the Sierpinski tetrahedron?

Thanks for listening!