1. Which of the points below is on the graph of the equation $x^3 - y^2 + 2xy + 12x = 5$?

   A. $(-1, -2)$

   B. $(1, -2)$

   C. $(2, 1)$

   D. $(-1, 2)$

   E. $(-2, -1)$

2. Solve for x: $\sqrt{25 + \sqrt{x}} = 6$.

   A. 64

   B. 81

   C. 100

   D. 121

   E. 144

3. A math book has 403 regularly numbered pages. On how many pages does the digit 5 appear in the page number?

   A. 76

   B. 77

   C. 78

   D. 79

   E. 80

4. Today is Thursday, October 8, 2015. What day of the week was August 3, 2014? (2014 and 2015 are not leap years and so contain 365 days. There are 31 days in August, 30 in September, and 31 in October.)

   A. Saturday

   B. Friday

   C. Tuesday

   D. Sunday

   E. Monday

5. For which ordered pairs of real numbers (a, b) is it true that $\ln(a + b) = \ln a + \ln b$?

   A. None.

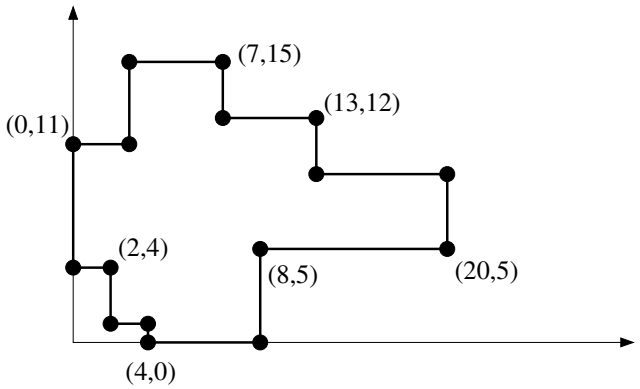
   B. All positive pairs (a, b).

   C. All positive pairs (a, b) with $a = b$.

   D. $(2, 2)$ only.

   E. All positive pairs (a, b) with $b = \dfrac{a}{a-1}$.

6. Find the perimeter of the shape shown below. (Assume all lines are vertical or horizontal.)



   A. 70

   B. 75

   C. 80

   D. 85

   E. 90

7. We define the "taxicab distance" between two points in the plane $P = (a, b)$ and $Q = (c, d)$ by the formula $\mathrm{TD}(P, Q) = |a - c| + |b - d|$. Then the set of all points which are taxicab distance 2 from the origin (0,0) forms

   A. A square.

   B. A line.

   C. A circle.

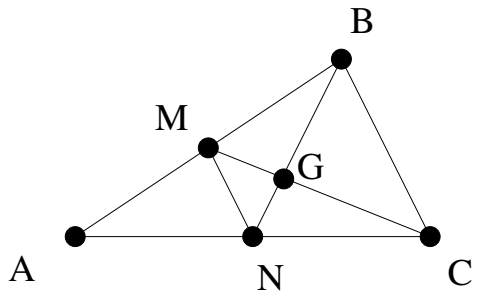
   D. Two lines.

   E. The empty set.

8. Any three members of the Cowboy painting team, working together, can paint eight rooms in four days. If you need twenty rooms painted in fifteen days, how many Cowboys should you hire?

   A. 1
   B. 2
   C. 3
   D. 5
   E. 8

9. Let ABC be a triangle with area 1. Let M and N be the midpoints of AB and AC, respectively, and let G be the intersection of MC and NB. Find the area of MNG.



   A. $\frac{1}{24}$
   B. $\frac{1}{12}$
   C. $\frac{1}{8}$
   D. $\frac{1}{6}$
   E. $\frac{1}{4}$

10. The functions f and g satisfy

$$f(x) + g(x) = 3x + 5$$
$$f(x) - g(x) = 5x + 7$$

    for all values of x. What is the value of $f(2)g(2)$?

   A. -42
   B. -24
   C. -6
   D. 12
   E. 30

11. Let $ABCD$ be a square with sides of length 20, and let E, F, G, and H be points on AB, BC, CD, and AD, respectively, such that $EFGH$ is a square with sides of length 15. Find $AG^2 + BG^2$.

    A. 825

    B. 875

    C. 925

    D. 975

    E. 1025

12. Your (small capacity) iPod contains ten songs by The Aardvarks, ten songs by The Binturongs, and five songs by The Cement Mixers. You listen to three songs in shuffle mode. What is the probability of hearing the same band at least twice? (Assume that shuffle mode chooses the songs at random, fairly and without replacement.)

    A. $\frac{7}{9}$

    B. $\frac{18}{23}$

    C. $\frac{121}{125}$

    D. $\frac{133}{138}$

    E. $\frac{563}{575}$

13. Ten lines are drawn in the plane, in such a way that no two lines are parallel and no three lines intersect at any one point. Into how many regions do these lines divide the plane?

    A. 11

    B. 23

    C. 56

    D. 277

    E. 1024

14. What is
$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{2014\cdot 2015}$$
as a fraction in lowest terms?

A. $\frac{4031}{4028}$
B. 1
C. $\frac{2014}{2015}$
D. $\frac{4031}{2015}$
E. $\frac{2016\cdot 4031}{6(2014!)}$

15. For which k is the vertex of the parabola $y=x^2+(kx-4)x+1$ on the line $y=0$?

A. All real k.
B. All positive k.
C. $k=3$ only.
D. All k on the interval $(0,4)$.
E. No real values of k.

16. How many diagonals does a convex decagon have? (A decagon has ten sides. The sides don't count as diagonals, so for example a square has two diagonals.)

A. 19
B. 23
C. 27
D. 31
E. 35

17. Let $f(x)$ be a function satisfying the equation $f\left(\dfrac{x}{1-x}\right) = \sqrt{x}$ for all x in the interval $(0, 1)$. If θ is an acute angle, find $f(\tan^2\theta)$.

A. $\sin\theta$

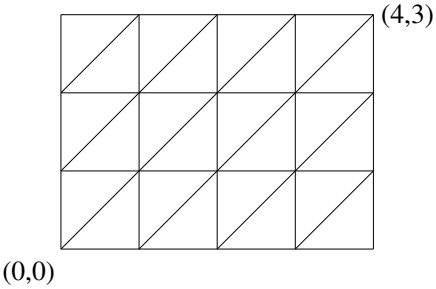
B. $\cos\theta$

C. $\cot\theta$

D. $\sec\theta$

E. $\csc\theta$

18. A robot begins at the origin, and wants to travel to the point $(4, 3)$ by making unit moves in the positive x or y directions, or diagonal moves, as in the diagram below. How many different paths can the robot take?



A. 120

B. 123

C. 126

D. 129

E. 132

19. Find x, if $x^{x^{x^{\cdot^{\cdot^{\cdot}}}}} = 2$. Assume a unique solution exists and is positive. (Towers of exponents are evaluated from top to bottom, so $x^{x^{x^{\cdot^{\cdot^{\cdot}}}}}$ means $x\text{^}(x\text{^}(x\text{^}(\ldots)))$.)

A. $\log_2 e$

B. $2^{\frac{1}{e}}$

C. $\sqrt{e}$

D. $\ln 2$

E. $\sqrt{2}$

20. A hall contains 2015 lockers, numbered 1 through 2015 in order, all open. A student starts at locker number 1 and runs down the hallway, closing every second locker (i.e., all the even-numbered ones.) Then she turns around and runs back, again closing every second locker (i.e, numbers 2013, 2009, etc.). She repeats this process until only one locker is left open. What is its number?

A. 139

B. 403

C. 651

D. 671

E. 1343

21. In how many zeros does the base ten representation of 2015! end?

A. 201

B. 403

C. 502

D. 648

E. 1081

22. Let $ABCD$ be an isosceles trapezoid with $AB = 4$, $BC = AD = 2$ and $\angle ABC = 120°$. If AC and BD intersect at E, find $\dfrac{BE}{DE}$.

A. 1

B. 2

C. $\dfrac{1}{2}$

D. $\dfrac{2}{3}$

E. $\dfrac{3}{2}$

23. How many ordered pairs of complex numbers (z, w) satisfy the two relations

$$z^2w + zw^2 = 30$$
$$zw + z + w = 11$$

A. 2

B. 3

C. 4

D. 6

E. 8

24. Let A be the sum of the digits of 2015^{2015} written in base 10. Let B be sum of the digits of A. What is the sum of the digits of B?

A. 1

B. 2

C. 5

D. 7

E. 8

25. Quadrilateral $ABCD$ has $AD = 2$, $BC = 1$, $CD = 5$, and $\angle A = \angle B = 60^\circ$. Find AB.

A. $\frac{9 + \sqrt{13}}{2}$

B. $\frac{7 + \sqrt{33}}{2}$

C. $\frac{5 + \sqrt{61}}{2}$

D. $\frac{3 + \sqrt{97}}{2}$

E. $\frac{1 + \sqrt{141}}{2}$