Math 2163

Jeff Mermin's section, Quiz 6, November 10

1. **(2 points each)** Indicate whether the following statements are true or false. ("True" means "Always true", "false" means "sometimes false" or "possibly false".) No justification is necessary. Write out the whole word "true" or "false".

   On these problems, x and y are the usual rectangular coordinates for $\mathbb{R}^2$, f and g are continuous functions, R is a closed and bounded region, and dA stands for $dxdy$, as it does in the text.

   (a) If $\iint_R f dA \geq \iint_R g dA$, then $f(x,y) \geq g(x,y)$ for all (x,y) in R.

   This is $\boxed{\text{false}}$. Suppose R is $[-1,2]\times[0,1]$, $f=x$, and $g=0$. Then $\iint_R f dA = 1$, $\iint_R g\, dA = 0$. But $f(-1,0)=-1$ and $g(-1,0)=0$.

   (b) If $f(x,y) \geq 0$ for all $(x,y) \in R$, then $\iint_R f dA \geq 0$.

   This is $\boxed{\text{true}}$. We have $\iint_R f\,dA = \lim \sum f(x_i,y_j)\Delta x \Delta y$, and $f(x_i,y_j) \geq 0$, $\Delta x > 0$, $\Delta y > 0$ so $f(x_i,y_j)\Delta x\Delta y \geq 0$. Thus the integral is a (limit of a) sum of positive terms.

   (c) $\iint_R f(x,y)g(x,y)dA = \left(\iint_R f(x,y)dA\right)\left(\iint_R g(x,y)dA\right)$.

   This is $\boxed{\text{false}}$. Almost any R, f, g will illustrate it. For example if R is $[-1,1]\times[-1,1]$ and $f=g=1$, then the LHS is 4 but the RHS is 16.

   (d) If $f(x,y) \geq g(x,y)$ for all (x,y) in R, then $\iint_R f dA \geq \iint_R g dA$.

   This is $\boxed{\text{true}}$. From (b) $\iint_R (f-g)\,dA \geq 0$.

   (e) If $F_1(x,y)$ and $F_2(x,y)$ are both antiderivatives of $f(x,y)$ with respect to x (that is, $\frac{\partial}{\partial x}(F_1) = \frac{\partial}{\partial x}(F_2) = f$), then $F_1(x,y) - F_2(x,y)$ is a constant function.

   This is $\boxed{\text{false}}$. $F_1 - F_2$ is a function that x thinks is constant, but it could behave interestingly with y.

   For example, maybe $f(x,y)=0$, $F_1(x,y)=y$, $F_2(x,y)=0$.

2. **(5 points)** Compute $\int_{x=-2}^{x=2}\int_{y=x^2}^{y=8-x^2} (16-2y)\,dydx$.

$$= \int_{x=-2}^{x=2} \left[16y - y^2\right]_{y=x^2}^{y=8-x^2} dx$$

$$= \int_{x=-2}^{x=2} \left[128 - 16x^2 - (64 - 16x^2 + x^4)\right] - \left[16x^2 - x^4\right] dx$$

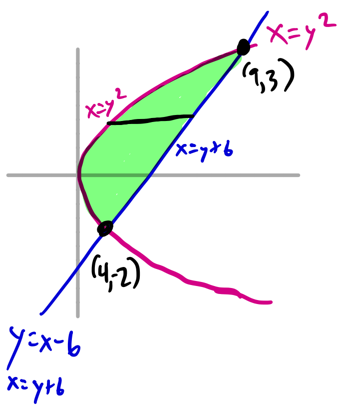
$$= \int_{x=-2}^{x=2} 64 - 16x^2 \, dx$$

$$= \left[64x - \frac{16}{3}x^3\right]_{x=-2}^{x=2}$$

$$= \left(128 - \frac{128}{3}\right) - \left(-128 - \frac{-128}{3}\right)$$

$$= \boxed{\frac{512}{3}}.$$

3. **(5 points)** Let D be the region bounded by the curves $x = y^2$ and $y = x - 6$. Express the double integral $\iint_D x + y\,dydx$ as an iterated integral. **Do not evaluate.**



After sketching the curves, we do some algebra to find the vertices: If $x=y^2$ and $y=x-6$, then $x=y+6$, so $y^2=y+6$, i.e. $y^2-y-6=0$, so $y=3$ ($x=9$) or $y=-2$ ($x=4$).

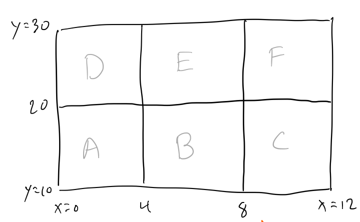
The y-boundaries are thus -2 and 3. For any given y, x ranges from y^2 to $y+6$. So our integral is

$$\int_{y=-2}^{y=3}\int_{x=y^2}^{x=y+6} x+y\,dy\,dx.$$

Extra Credit **(4 points)**: Some values of a continuous function $f(x,y)$ on the rectangle $R = \{0 \le x \le 12, 10 \le y \le 30\}$ are given in the table below. (Apparently f is hard to compute, because some values are unknown). Estimate the value of $\iint_R f(x,y)dA$ using a Riemann sum with at least six summands.

Warning: The $x = 8$ column is missing.

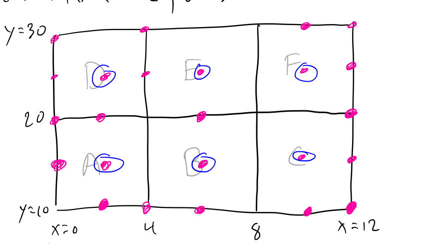
Start by chopping R into six pieces:



(There are lots of ways to chop, but this feels easiest)

y \ x	0	2	4	6	10	12
10	?	−10	−10	−9	−6	0
15	−7	−4	?	2	4	8
20	−1	3	?	8	?	11
25	−3	−1	−1	−1	0	?
30	7	?	10	?	11	13

Then mark the points with known data:



And choose one from each region.
(Again, there are many choices, but the centers seem good.)

Now, build a table:

Region	P	f(P)	dx	dy	f dxdy
A	(2,15)	−4	4−0=4	20−10=10	−4(4)(10) = −160
B	(6, 15)	2	8−4=4	20−10=10	2(4)(10) = 80
C	(10,15)	4	12−8=4	20−10=10	4(4)(10) = 160
D	(2,25)	−1	4−0=4	30−20=10	−1(4)(10) = −40
E	(6,25)	−1	8−4=4	30−20=10	−1(4)(10) = −40
F	(10,25)	0	12−8=4	30−20=10	0(4)(10) = 0

So our approximation is $\boxed{-4(4)(10) + 2(4)(10) + 4(4)(10) + -1(4)(10) + (-1)(4)(10) + 0(4)(10)}$

$= 0.$