Math 2163

Jeff Mermin's section, Quiz 2, September 1

1. **(2 points each)** Indicate whether the following statements are true or false. ("True" means "Always true", "false" means "sometimes false" or "possibly false".) No justification is necessary. Write out the whole word "true" or "false".

   On these problems, a, b, c, p, q, and r are real numbers, P is a point, t is a variable, $\mathbf{v}$ and $\mathbf{w}$ are (nonzero) vectors, and x, y, and z are the usual rectangular coordinates for $\mathbb{R}^3$

   (a) If $\mathbf{v}$ is a direction vector for a line ℓ, then $2\mathbf{v}$ is also a direction vector for ℓ.

   This is true.

   (b) The line $(x, y, z) = (p, q, r) + t\langle a, b, c\rangle$ is contained in the plane with equation $a(x - p) + b(y - q) + c(z - r) = 0$.

   This is false. $\langle a, b, c\rangle$ is normal to the plane, but parallel to the line.

   (c) $\mathbf{v}$ and $\mathbf{w}$ are perpendicular if and only if $\mathbf{v} \bullet \mathbf{w} = 0$.

   This is true.

   (d) The equations $(x, y, z) = (P + t\mathbf{v})$ and $(x, y, z) = P - t\mathbf{v}$ define the same line.

   This is true

   (e) The planes $x + y + z = 2$ and $2x + 2\cancel{z}y + 2z = 1$ are parallel.

   This is true. The second plane is $x+y+z = \frac{1}{2}$; $\langle 1, 1, 1\rangle$ is normal to both.

2. **(4 points)** Compute the cross product

$$\langle 4, 0, -3\rangle \times \langle 0, 4, 2\rangle$$

$$= \langle (0)(2) - (-3)(4), (-3)(0) - (4)(2), (4)(4) - (0)(0)\rangle$$

$$= \boxed{\langle 12, -8, 16\rangle}$$

Most of the mnemonics and procedures for the cross product can be easily misunderstood in a way that gets the second term off by a sign. That's why it's worth four points instead of three.

If you're making that mistake, you want to act to correct it <u>now</u>, before the incorrect thing becomes (more) ingrained.

3. **(6 points)** Find the equation for the plane that contains the points $P = (3, -1, 5)$, $Q = (4, -2, -3)$, and $R = (0, 1, -2)$

First, compute two direction vectors within the plane.

These will be $\pm\overrightarrow{PQ} = \langle 1, -1, -8 \rangle$,

$\pm\overrightarrow{PR} = \langle -3, 2, -7 \rangle$,

or $\pm QR = \langle -4, 3, 1 \rangle$.

Now take the cross product of any two of these; that's a normal vector $n = \overrightarrow{PQ} \times \overrightarrow{PR} = \langle (-1)(-7) - (-8)(2), (-8)(-3) - (1)(-7), (1)(2) - (-1)(-3) \rangle$

$= \langle 23, 31, -1 \rangle$

The equation of the plane is thus

$23x + 31y - z = D$, where we get D by plugging in $P, Q,$ or R

(e.g., using P, $D = 23(3) + 31(-1) - 1(5) = 33$)

So our equation is $\boxed{23x + 31y - z = 33}$

(If there's time, check with the other points!
$23(4) + 31(-2) - (-3) = 33$
$23(0) + 31(1) - (-2) = 33$)

Alternatively, $23(x-3) + 31(y+1) - (z-5) = 0$
or $23(x-4) + 31(y+2) - (z+3) = 0$
or $23(x-0) + 31(y-1) - (z+2) = 0$.

Extra Credit **(3 points)**: Suppose we know that $\mathbf{v} \times \mathbf{w} = \langle 1, 2, 3 \rangle$. Compute the following, or explain why we need more information.

(a) $(\mathbf{v} + \mathbf{w}) \times (\mathbf{v} - \mathbf{w})$. $= v \times (v - w) + w \times (v - w)$
$= v \times v - v \times w + w \times v - w \times w$
$= 0 - v \times w - v \times w - 0$
$= -2(v \times w) = \boxed{\langle -2, -4, -6 \rangle}$.

(b) $(\mathbf{v} + \mathbf{w}) \bullet (\mathbf{v} - \mathbf{w})$ $= v \bullet (v - w) + w \bullet (v - w)$
$= v \bullet v - v \bullet w + w \bullet v - w \bullet w$
$= v \bullet v - v \bullet w + v \bullet w - w \bullet w$
$= v \bullet v + (\quad 0 \quad) - w \bullet w$.

And now we're stuck, since we have (essentially) no information about $v \bullet v$ or $w \bullet w$.

(c) $(\mathbf{v} - \langle 1, 1, 1 \rangle) \times (\langle 1, 1, 1 \rangle - \mathbf{v})$
$= (v - \langle 1, 1, 1 \rangle) \times -(v - \langle 1, 1, 1 \rangle)$
$= 0$

(If $u = v - \langle 1, 1, 1 \rangle$, this is $u \times -u$
$= -(u \times u)$
$= -0$.