Math 2163

Jeff Mermin's section, Quiz 1, August 25

1. **(2 points each)** Indicate whether the following statements are true or false. ("True" means "Always true", "false" means "sometimes false" or "possibly false".) No justification is necessary. Write out the whole word "true" or "false".

   On these problems, a and b are numbers, $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are vectors in $\mathbb{R}^3$, and x, y, and z are the usual coordinates in $\mathbb{R}^3$.

   (a) $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$.

   This is $\boxed{\text{true.}}$

   (b) $\mathbf{u} \bullet \mathbf{v} = \mathbf{v} \bullet \mathbf{u}$.

   This is $\boxed{\text{true.}}$

   (c) $|\mathbf{u} \bullet \mathbf{v}| \leq \|\mathbf{v}\| \, \|\mathbf{w}\|$

   This is meant to be $\boxed{\text{true}}$ (since $|u \bullet v| = \|u\| \, \|v\| \, |\cos\theta| \leq \|u\| \, \|v\| (1)$), but the typo makes it $\boxed{\text{false}}$.

   (d) The equations $x = 2$, $y = -1$, $z = 0$ define a line in $\mathbb{R}^3$.

   This is $\boxed{\text{false}}$ They define the point $(2,-1,0)$, which is not a line.

   (e) $(a + b)\mathbf{u} = a\mathbf{u} + b\mathbf{u}$.

   This is $\boxed{\text{true.}}$

2. **(4 points)** Find a (vector) equation describing the line through $P = (4, 5, 0)$ and $Q = (-3, 4, 0)$.

   The direction vector is $\overrightarrow{PQ} = (-3,4,0) - (4,5,0)$

   $= \langle -7, -1, 0 \rangle$;

   We can factor out a -1 and use $\langle 7, 1, 0 \rangle$ instead.

   Our equation is $\boxed{(x,y,z) = (4,5,0) + \langle 7,1,0 \rangle t}$

   $(x,y,z) = (-3,4,0) + \langle 7,1,0 \rangle t$

   or $(x,y,z) = (4,5,0)t + (-3,4,0)(1-t)$

   are also correct (and reasonably likely) answers.

3. **(2 points)** Find two points on the line with vector equation

$$(x, y, z) = (2, -1, 1) + \langle -2, -2, 4 \rangle t.$$

• Plug in $t = 0$ (or read off P from $(x, y, z) = P + t\mathbf{v}$): $\boxed{(2, -1, 1).}$

• Plug in $t = 1$: $(2, -1, 1) + \langle -2, -2, 4 \rangle = \boxed{(0, -3, 5)}$

(or plug in any other value(s) for t to get more points. e.g., $t = -1$ gives $(4, 1, -3)$, $t = 2$ gives $(-2, -5, 9)$.)

4. **(4 points)** Let $\mathbf{v} = \langle 1, -1, 3 \rangle$ and $\mathbf{w} = \langle 1, 2, 5 \rangle$.

(a) Compute $\mathbf{v} \bullet \mathbf{w}$.

$$\begin{aligned} v \bullet w &= (1)(1) + (-1)(2) + (3)(5) \\ &= 1 - 2 + 15 \\ &= \boxed{14} \end{aligned}$$

(b) Is the angle between $\mathbf{v}$ and $\mathbf{w}$ acute, right, or obtuse?

$v \bullet w = 14$ is positive, so the angle is $\boxed{\text{acute}}$.

Extra Credit **(3 points)**: Do the lines

$$\ell : (x, y, z) = (1, -2, 5) + \langle 1, 4, 1 \rangle t$$

and

$$m : (x, y, z) = (-3, 5, 5) + \langle -1, -3, 3 \rangle t$$

intersect? If so, what is their point of intersection?

If they intersect, they intersect at some point $P = (x, y, z)$.

Since ℓ passes through P, we know $P = (1, -2, 5) + \langle 1, 4, 1 \rangle t$ for some time t.

Since m passes through P, we know $P = (-3, 5, 5) + \langle -1, -3, 3 \rangle s$ for some time s (probably not equal to t).

We get six equations in five variables:

$$\underbrace{\begin{aligned} x &= 1 + t \\ y &= -2 + 4t \\ z &= 5 + t \end{aligned}}_{\text{since } P \text{ is on } \ell} \qquad \underbrace{\begin{aligned} x &= -3 - s \\ y &= 5 - 3s \\ z &= 5 + 3s \end{aligned}}_{\text{since } P \text{ is on } m}$$

Looking for a simultaneous solution, we substitute out x, y, z:

$$\begin{aligned} 1 + t &= -3 - s \quad (A) \\ -2 + 4t &= 5 - 3s \quad (B) \\ 5 + t &= 5 + 3s \quad (C). \end{aligned}$$

From (C), $t = 3s$.
From (A), $1 + 3s = -3 - s$, so $s = -1$.
From (B), $-2 + 12s = 5 - 3s$, so $s = \frac{7}{15} \neq -1$.

Thus there's no solution, so the lines $\boxed{\text{don't intersect.}}$