Math 2163

Jeff Mermin's section, Quiz 6, November 10

1. **(2 points each)** Indicate whether the following statements are true or false. ("True" means "Always true", "false" means "sometimes false" or "possibly false".) No justification is necessary. Write out the whole word "true" or "false".

   On these problems, x and y are the usual rectangular coordinates for $\mathbb{R}^2$, f and g are continuous functions, R is a closed and bounded region, and dA stands for $dxdy$, as it does in the text.

   (a) If $\iint_R f dA \geq \iint_R g dA$, then $f(x,y) \geq g(x,y)$ for all (x,y) in R.

   (b) If $f(x,y) \geq 0$ for all $(x,y) \in R$, then $\iint_R f dA \geq 0$.

   (c) $\iint_R f(x,y)g(x,y)dA = \left(\iint_R f(x,y)dA\right)\left(\iint_R g(x,y)dA\right)$.

   (d) If $f(x,y) \geq g(x,y)$ for all (x,y) in R, then $\iint_R f dA \geq \iint_R g dA$.

   (e) If $F_1(x,y)$ and $F_2(x,y)$ are both antiderivatives of $f(x,y)$ with respect to x (that is, $\frac{\partial}{\partial x}(F_1) = \frac{\partial}{\partial x}(F_2) = f$), then $F_1(x,y) - F_2(x,y)$ is a constant function.

2. **(5 points)** Compute $\int_{x=-2}^{x=2} \int_{y=x^2}^{y=8-x^2} (16 - 2y)\, dydx$.

3. **(5 points)** Let D be the region bounded by the curves $x = y^2$ and $y = x - 6$. Express the double integral $\iint_D x + y \, dydx$ as an iterated integral. **Do not evaluate**.

Extra Credit **(4 points)**: Some values of a continuous function $f(x, y)$ on the rectangle $R = \{0 \leq x \leq 12, 10 \leq y \leq 30\}$ are given in the table below. (Apparently f is hard to compute, because some values are unknown). Estimate the value of $\iint_R f(x, y) dA$ using a Riemann sum with at least six summands.

Warning: The $x = 8$ column is missing.

y \ x	0	2	4	6	10	12
10	?	−10	−10	−9	−6	0
15	−7	−4	?	2	4	8
20	−1	3	?	8	?	11
25	−3	−1	−1	−1	0	?
30	7	?	10	?	11	13