Math 2163

Jeff Mermin's section, Quiz 5, October 20

1. **(2 points each)** Indicate whether the following statements are true or false. ("True" means "Always true", "false" means "sometimes false" or "possibly false".) No justification is necessary. Write out the whole word "true" or "false".

   On these problems, a, b, and c are numbers, x, y, and z are the usual rectangular coordinates, $f = f(x, y)$ and $F = F(x, y, z)$ are smoothly differentiable multivariable functions, and P is a point.

   (a) $\nabla f(a, b)$ is normal to the level curve of $f(x, y)$ passing through (a, b).

   (b) If D is neither closed nor bounded, then f fails to have a global maximum on D.

   (c) If $\mathbf{u}$ is the unit vector in the direction of ∇F at P, then $D_{\mathbf{u}}(F)(P) = \|\nabla F\|$.

   (d) $\nabla F(a, b, c)$ is tangent to the surface $F(x, y, z) = 0$ at the point (a, b, c).

   (e) If f has two local maxima, then it must have a local minimum.

2. **(4 points)** Find the equation for the tangent plane to the level surface of $F(x, y, z) = x^2 + y^2 + z^2 - 3xyz$ at the point $P = (2, 5, 7)$.

3. **(4 points)** Consider the function $f(x, y) = x^3 + 9xy - 9y^2 - 21x$. You may assume that

$$f_x = 3x^2 + 9y - 21 \qquad f_y = 9x - 18y$$

$$f_{xx} = 6x \qquad f_{xy} = 9 \qquad f_{yy} = -18$$

Decide whether the points below are critical points of f. Then, if they are, classify them as local maxima, local minima, or saddle points.

(a) $P = (0, 0)$.

(b) $Q = (1, 2)$.

(c) $S = (2, 1)$.

4. **(2 points)** Does the function f above have any other critical points? Find them, or verify that they are all listed above.

Extra Credit **(2 points)**: Sketch the contour map of the function f above. (This will probably require checking the type(s) of any critical points you found in problem 4.)