Math 2163

Jeff Mermin's section, Quiz 2, September 1

1. **(2 points each)** Indicate whether the following statements are true or false. ("True" means "Always true", "false" means "sometimes false" or "possibly false".) No justification is necessary. Write out the whole word "true" or "false".

   On these problems, $\mathbf{r} = \mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$ and $\mathbf{s} = \mathbf{s}(t)$ are parametric curves in $\mathbb{R}^3$, t is a parameter, and $\mathbf{v}$ and $\mathbf{w}$ are (nonzero) vectors in $\mathbb{R}^3$.

   (a) $\dfrac{d}{dt}(\mathbf{r} \bullet \mathbf{s}) = \left(\dfrac{d\mathbf{r}}{dt} \bullet \mathbf{s}\right) + \left(\mathbf{r} \bullet \dfrac{d\mathbf{s}}{dt}\right)$.

   (b) $\dfrac{d}{dt}(\mathbf{r} \bullet \mathbf{s}) = \dfrac{d\mathbf{r}}{dt} \bullet \dfrac{d\mathbf{s}}{dt}$.

   (c) $\dfrac{d}{dt}(\mathbf{r} \times \mathbf{s}) = \dfrac{d\mathbf{r}}{dt} \times \dfrac{d\mathbf{s}}{dt}$.

   (d) $\displaystyle\int_{t=0}^{t=1} \frac{d\mathbf{r}}{dt} dt = \mathbf{r}(1) - \mathbf{r}(0)$.

   (e) $\mathbf{v}$ and $\mathbf{w}$ are perpendicular if and only if $\mathbf{v} \times \mathbf{w} = \mathbf{0}$.

2. **(5 points)** Find the point of intersection between the line $\ell : (x, y, z) = (-1, -3, 5) + \langle -2, 2, 0 \rangle t$ and the plane $L : 3x - y + 3z = -25$, or explain why no such point exists.

3. **(5 points)** Consider the semidouble quasihelix C parametrized by

$$C : \mathbf{r}(t) = (\cos \pi t, \sin \pi t, t^2)$$

and the point $P = \left(0, 1, \frac{1}{4}\right)$.

(a) Show that P is on C.

(b) Find the equation for the tangent line to C at P.

Extra Credit **(3 points)**: An object in space experiences an acceleration given by the function $\mathbf{a}(t) = \langle t^2, 2t, 1 - t^3 \rangle$. It passes through the points $(0, 0, 0)$ and $(1, 2, 3)$ at $t = 0$ and $t = 1$, respectively. Where is it at $t = 2$?