

1.

- (a) The sides have equations  $x + y = \pm 1$  and  $x - y = \pm 1$ . Their lengths are  $\sqrt{2}$ . The area of the square is 2.

- (b) The area can be expressed as

$$\int_{x=-1}^{x=0} \int_{y=-1-x}^{y=1+x} dydx + \int_{x=0}^{x=1} \int_{y=x-1}^{y=1-x} dydx.$$

- (c) The area is

$$\int_{y=-1}^{y=0} \int_{x=-1-y}^{x=1+y} dx dy + \int_{y=0}^{y=1} \int_{x=y-1}^{x=1-y} dx dy.$$

- (d) You should get 4.

2.

- (a) The sides have equations  $3x + 4y = \pm 33$  and  $12x + 5y = \pm 33$ . The area is 132.

- (b) You should get

$$\int_{x=-9}^{x=-1} \int_{y=\frac{-33-12x}{5}}^{y=\frac{33-3x}{4}} dy dx + \int_{x=-1}^{x=1} \int_{y=\frac{-33-12x}{5}}^{y=\frac{33-12x}{5}} dy dx + \int_{x=1}^{x=9} \int_{y=\frac{-33-3x}{4}}^{y=\frac{33-12x}{5}} dy dx.$$

- (c) You should get

$$\int_{y=-15}^{y=-9} \int_{x=\frac{-33-4y}{3}}^{x=\frac{33-5y}{12}} dx dy + \int_{y=-9}^{y=9} \int_{x=\frac{-33-5y}{12}}^{x=\frac{33-5y}{12}} dx dy + \int_{y=9}^{y=15} \int_{x=\frac{-33-5y}{12}}^{x=\frac{33-4y}{3}} dx dy.$$

- (d)

- (e) Try  $u = 3x + 4y$  and  $v = 12x + 5y$ . Then the area is

$$\int_{u=-33}^{u=33} \int_{v=-33}^{v=33} dx dy.$$

- (f) You should get  $66^2 = 4356$ , which is off by a factor of 33.

- (g) You should get  $\frac{4356}{65}$ , which is definitely not  $\frac{4356}{33}$ .

3.

- (a)  $e^x \sin x[\mathbf{dx}] + e^x \cos x[\mathbf{dx}]$ .
- (b)  $\ln x[\mathbf{dx}] + [\mathbf{dx}]$ .
- (c)  $e^x \sin x \ln x[\mathbf{dx}] + e^x \sin x[\mathbf{dx}] + xe^x \ln x \sin x[\mathbf{dx}] + xe^x \ln x \cos x[\mathbf{dx}]$ .
- (d)  $e^x \sin x \ln x[\mathbf{dx}] + e^x \sin x[\mathbf{dx}] + xe^x \ln x \sin x[\mathbf{dx}] + xe^x \ln x \cos x[\mathbf{dx}]$ .  
(What does the product rule say?)
- (e) At this stage, the best we can do is

$$e^x \sin x \ln x([\mathbf{dx}])^2 + e^x \sin x([\mathbf{dx}])^2 + e^x \cos x \ln x([\mathbf{dx}])^2 + e^x \cos x([\mathbf{dx}])^2.$$

4.

- (a)  $2x[\mathbf{dx}] + 2y[\mathbf{dy}] - y[\mathbf{dx}] - x[\mathbf{dy}]$ .
- (b)  $ye^z[\mathbf{dx}] + xe^z[\mathbf{dy}] + xy e^z[\mathbf{dz}]$ .
- (c)  $3x^2ye^z[\mathbf{dx}] + x^3e^z[\mathbf{dy}] + x^3ye^z[\mathbf{dz}] + y^3e^z[\mathbf{dx}] + 3xy^2e^z[\mathbf{dy}] + xy^3e^z[\mathbf{dz}] - x^2y^2e^z[\mathbf{dz}]$ .
- (d)  $3x^2ye^z[\mathbf{dx}] + x^3e^z[\mathbf{dy}] + x^3ye^z[\mathbf{dz}] + y^3e^z[\mathbf{dx}] + 3xy^2e^z[\mathbf{dy}] + xy^3e^z[\mathbf{dz}] - x^2y^2e^z[\mathbf{dz}]$ .
- (e) At this stage, the best we can do is

$$\begin{aligned} & 2xye^z([\mathbf{dx}])^2 + 2x^2e^z[\mathbf{dx}][\mathbf{dy}] + 2x^2ye^z[\mathbf{dx}][\mathbf{dz}] \\ & + 2y^2e^z[\mathbf{dy}][\mathbf{dx}] + 2xye^z([\mathbf{dy}])^2 + 2xy^2e^z[\mathbf{dy}][\mathbf{dz}] \\ & + y^2e^z([\mathbf{dx}])^2 + xy e^z[\mathbf{dx}][\mathbf{dy}] + xy^2e^z[\mathbf{dx}][\mathbf{dz}] \\ & + xye^z[\mathbf{dy}][\mathbf{dx}] + x^2e^z([\mathbf{dy}])^2 + x^2ye^z[\mathbf{dy}][\mathbf{dz}]. \end{aligned}$$

5.

- (a) Use the fact that  $[\mathbf{df}][\mathbf{df}] = -[\mathbf{df}][\mathbf{df}]$ .
- (b) 0.
- (c)  $2x^2e^z[\mathbf{dx}][\mathbf{dy}] - 2y^2e^z[\mathbf{dx}][\mathbf{dy}] - xye^z[\mathbf{dx}][\mathbf{dy}] + xye^z[\mathbf{dx}][\mathbf{dy}] + 2x^2ye^z[\mathbf{dx}][\mathbf{dz}] + xy^2e^z[\mathbf{dx}][\mathbf{dz}] + 2xy^2e^z[\mathbf{dy}][\mathbf{dz}] + x^2ye^z[\mathbf{dy}][\mathbf{dz}]$ .
- (d)  $2[\mathbf{dy}][\mathbf{dx}] = -2[\mathbf{dx}][\mathbf{dy}]$ .
- (e) 33 $[\mathbf{dy}][\mathbf{dx}]$ .

6.

(a)

$$[\mathbf{dx}] = \cos \theta [\mathbf{dr}] - r \sin \theta [\mathbf{d}\theta]$$
$$[\mathbf{dy}] = \sin \theta [\mathbf{dr}] + r \cos \theta [\mathbf{d}\theta].$$

(b)  $r[\mathbf{dr}][\mathbf{d}\theta]$ .

(c)  $-r[\mathbf{dr}][\mathbf{d}\theta]$ .

(d) Movement in the direction of  $dx$  followed by  $dy$  is a left turn, as is movement in the direction of  $dr$  followed by  $d\theta$ . Movement in the direction of  $dy$  then  $dx$ , or  $d\theta$  then  $dr$ , is a right turn.

(e) A likely intermediate step is the two formulas

$$[\mathbf{dr}] = \frac{x}{r} [\mathbf{dx}] + \frac{y}{r} [\mathbf{dy}]$$
$$[\mathbf{d}\theta] = \frac{\cos^2 \theta}{x} [\mathbf{dy}] - \frac{y \cos^2 \theta}{x^2} [\mathbf{dx}]$$

7.

(a)

(b) Use example 6 directly to change  $[\mathbf{dx}][\mathbf{dy}]$  to  $r[\mathbf{dr}][\mathbf{d}\theta]$ . Then use part (a) to finish.

(c)

(d)

$$[\mathbf{dx}] = \sin \phi \cos \theta [\mathbf{d}\rho] + \rho \cos \phi \cos \theta [\mathbf{d}\phi] - \rho \sin \phi \sin \theta [\mathbf{d}\theta]$$
$$[\mathbf{dy}] = \sin \phi \sin \theta [\mathbf{d}\rho] + \rho \cos \phi \sin \theta [\mathbf{d}\phi] + \rho \sin \phi \cos \theta [\mathbf{d}\theta]$$
$$[\mathbf{dz}] = \cos \phi [\mathbf{d}\rho] - \rho \sin \phi [\mathbf{d}\phi]$$

8.

(a)  $x - y = 0$ ,  $x - y = 3$ ,  $4y - x = 0$ , and  $4y - x = 9$ .

(b)  $u = 0$ ,  $u = 3$ ,  $v = 0$ , and  $v = 9$ .

(c)  $[\mathbf{du}][\mathbf{dv}] = 3[\mathbf{dy}][\mathbf{dx}]$

(d) “ $\int_{u=0}^{u=3} \int_{v=0}^{v=9}$ ”

(e) “ $e^{5u+v} (\frac{1}{3} dv du)$ ”

(f)  $\frac{1}{15} (e^{24} - e^{15} - e^9 + 1)$ .

9.

- (a)  $u = 1, u = 4, v = 1$ , and  $v = 4$ .
- (b)  $[\mathbf{d}\mathbf{u}][\mathbf{d}\mathbf{v}] = 2\frac{y}{x}[\mathbf{d}\mathbf{x}][\mathbf{d}\mathbf{y}]$ .
- (c) “ $\int_{v=1}^{v=4} \int_{u=1}^{u=4}$ ”.
- (d) “ $\left(\frac{u}{v} + uv\right) \left(\frac{1}{2v} dudv\right)$ ”.
- (e)  $\frac{225}{16}$ .

10.

- (a)  $u = \pm 3, v = 1$ , and  $v = 4$ .
- (b)  $[\mathbf{d}\mathbf{u}][\mathbf{d}\mathbf{v}] = 2(x^2 + y^2)[\mathbf{d}\mathbf{y}][\mathbf{d}\mathbf{x}]$ .
- (c) “ $\int_{u=-3}^{u=3} \int_{v=1}^{v=4}$ ”.
- (d) “ $\frac{v}{2} dv du$ ”.
- (e)  $\frac{45}{2}$ .