Math 2163

Jeff Mermin's section, Test 3, November 19

On the essay questions (# 2–4) write legibly in complete sentences, in such a way that I can easily tell what you are doing and why.

Put away your calculators, cell phones, smart watches, and other internet-enabled devices for the duration of the exam. If any of these are out for any reason, it will constitute an academic integrity violation.

14-week grade estimate

Category	Score	Possible
Exam 1		150
Exam 2		150
Exam 3		150
WebAssign		138
Written homework		40
Quizzes		100
Total		728

1. (**30 points**)Indicate whether the following statements are true or false. ("True" means "Always true", "false" means "sometimes false".) No justification is necessary on this problem. **Write the entire word "True" or "False"**. Illegible or abbreviated answers will receive no credit.

   In the statements below, a, b, c, d, d', p, q, and r are numbers, x, y, and z are the usual rectangular coordinates for $\mathbb{R}^3$, $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are vectors in $\mathbb{R}^3$, $f = f(x, y)$ and $g = g(x, y, z)$ are smoothly differentiable functions, D is a closed and bounded region inside $\mathbb{R}^2$, R is a closed and bounded region inside $\mathbb{R}^3$, and dV means what it means in the book.

   (a) There are functions $h_1(x)$ and $h_2(y)$ such that $f(x, y) = h_1(x)+h_2(y)$.

   (b) The line with equation $(x, y, z) = (p, q, r) + \langle a, b, c\rangle$ is contained in the plane with equation $a(x - p) + b(y - q) + c(z - r)$.

   (c) If $R = \{(x, y, z) : x^2 + y^2 + z^2 \leq 16\}$, then $\iiint_R dV = 36\pi$.

   (d) $(\mathbf{u} \times \mathbf{v}) \times \mathbf{w} = \mathbf{u} \times (\mathbf{v} \times \mathbf{w})$.

   (e) If R_1 and R_2 are disjoint regions and $R = R_1 \cup R_2$ is their union, then $\iiint_R g(x, y, z)dV = \iiint_{R_1} g(x, y, z)dV + \iiint_{R_2} g(x, y, z)dV$.

   (f) If f has an absolute maximum on the region D at (a, b), then f has a local maximum at (a, b).

   (g) $\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)$.

   (h) If ℓ is the line through $P = (1, 2, 3)$ and $Q = (4, 5, 6)$, then $\langle 4, 5, 6\rangle$ is a direction vector for ℓ.

   (i) $\iiint_R f(x, y, z)dxdydz = \iiint_R f(x, y, z)dzdydx$.

   (j) The distance between the planes $F : ax + by + cz = d$ and $G : ax + by + cz = d'$ is $|d - d'|$.

2. (**30 points**) Consider the double integral $I = \displaystyle\int_{y=1}^{y=e} \int_{x=(\ln y)^2}^{x=\ln y} \frac{1}{\ln y}\, dx dy.$

(a) Rewrite I with the x-integral on the outside and the y-integral on the inside.

(b) Evaluate I, in whichever order makes more sense. [The integral tables are attached at the back of the exam. You **probably do** want to take advantage of them; if you do, indicate which entry you're using.]

3. (**15 points**) Some values of a function $f(x, y)$ are given in the table. Use a Riemann sum with at least six summands to estimate the value of $\int_{x=0}^{x=2} \int_{y=1}^{y=2} f(x, y)dydx.$

$y \backslash x$	0	0.5	1	1.5	2
0	1	1.4	1.8	2.2	2.6
.5	0.8	1.2	1.5	1.8	2.2
1	0.7	1	1.3	1.6	1.8
1.5	0.6	0.9	1	1.3	1.6
2	0.5	0.8	1	1.2	1.4

4. (**75 points**) Express each of the following as an iterated integral. You may use any coordinate system that makes sense, provided that you define anything other than rectangular, polar, or spherical coordinates. **Do not evaluate your integrals.**

   (a) The volume of the region R inside the sphere $x^2 + y^2 + z^2 = 25$ and above the plane $z = 3$.

(b) The volume of the region bounded by the surfaces $z = 1 - y^2$, $y = x^2$, and $z = 0$.

(c) $\displaystyle\iint_R \left((x-2y)\sin(2x+y)\right)^2 \, dxdy$, where R is the square with vertices $(\pi, 0)$, $(3\pi, \pi)$, $(2\pi, 3\pi)$, and $(0, 2\pi)$.

5. (**Extra credit: 20 points**) Let's explore the sizes of spheres in various dimensions. For the integrals, you may use any coordinate system you like so long as you're clear what you're doing. (I recommend polar coordinates or something analogous).

   (a) Use an integral to compute the length of the "unit line segment" $\{x : x^2 \leq 1\}$. Derive a formula for the length of a segment of radius r.

   (b) Use a double integral to compute the area of the "unit circle" $\{(x, y) : x^2 + y^2 \leq 1\}$. Derive a formula for the area of a circle of radius r.

   (c) Use a triple integral to compute the volume of the "unit sphere" $\{(x, y, x) : x^2 + y^2 + z^2 \leq 1\}$. Derive a formula for the volume of a sphere of radius r.

   (d) Use a quadruple integral to compute the hyper-volume of the "unit hypersphere" $\{(w, x, y, x) : w^2 + x^2 + y^2 + z^2 \leq 1\}$. Derive a formula for the hyper-volume of a hypersphere of radius r.