Math 2163

Jeff Mermin's section, Test 2, October 15

On the essay questions (# 2–6) write legibly in complete sentences, in such a way that I can easily tell what you are doing and why.

Put away your calculators, cell phones, smart watches, and other internet-enabled devices for the duration of the exam. If any of these are out for any reason, it will constitute an academic integrity violation.

10-week grade estimate

Category	**Score**	**Possible**
Exam 1		150
Exam 2		150
WebAssign		86
Written homework		20
Quizzes		62
Total		468

1. (**30 points**)Indicate whether the following statements are true or false. ("True" means "Always true", "false" means "sometimes false".) No justification is necessary on this problem. **Write the entire word "True" or "False"**. Illegible or abbreviated answers will receive no credit.

   In the statements below, a, b, and c are numbers, x, y, and z are the usual rectangular coordinates for $\mathbb{R}^3$, t is a parameter, $f = f(x, y)$ is a smoothly differentiable function, $P = (p, q, r)$ is a point in $\mathbb{R}^3$, $\mathbf{r} = \mathbf{r}(t) = \langle x(t), y(t), z(t)\rangle$ is a smooth curve in $\mathbb{R}^3$, and $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are vectors.

   (a) $\mathbf{v} \bullet \mathbf{w} = \mathbf{w} \bullet \mathbf{v}$.

   (b) $\mathbf{v}$ and $-2\mathbf{v}$ are parallel.

   (c) The equations $x = 2$, $y = 1$, $z = 0$ define a plane in $\mathbb{R}^3$.

   (d) $(\mathbf{u} \bullet \mathbf{v})\mathbf{w} = \mathbf{u}(\mathbf{v} \bullet \mathbf{w})$.

   (e) If f has two local maxima, then it must have a local minimum.

   (f) The equations $(x, y, z) = P + t\mathbf{v}$ and $(x, y, z) = P - t\mathbf{v}$ define the same line.

   (g) The line with equation $(x, y, z) = (p, q, r) + \langle a, b, c\rangle$ is contained in the plane with equation $a(x - p) + b(y - q) + c(z - r) = 0$.

   (h) If two planes in $\mathbb{R}^3$ intersect, they intersect in a line.

   (i) If $f_{xx} = f_{yy} = 0$, then the graph of $z = f(x, y)$ is a plane in $\mathbb{R}^3$.

   (j) $\displaystyle\int_{t=0}^{t=1} \frac{d\mathbf{r}}{dt} dt = \mathbf{r}(1) - \mathbf{r}(0)$.

2. (**30 points**) Compute all four of the second partial derivatives for

$$f(x, y) = \ln(x^2 - y^2) - \ln(x - y).$$

3. (**20 points**) Find an equation for the tangent plane to the graph of the function $f(x, y) = e^{xy}$ at the point $(1, 2, e^2)$.

4. (**20 points**) Find an equation for the tangent plane to the surface $x^2y + y^2z + z^2x = 2$ at the point $(0, 1, 2)$.

5. (**20 points**) Find the directional derivative in the direction of the vector $\langle 2, 3, 6\rangle$ for the function $F(x, y, z) = x^2 + y^2 - 2z^2$.

6. (**30 points**) Consider the function $f(x,y) = 8x - x^2 - 2xy^2$. You may assume that

$$f_x = 8 - 2x - 2y^2 \qquad f_y = -4xy$$

$$f_{xx} = -2 \qquad f_{xy} = -4y \qquad f_{yy} = -4x$$

Decide whether the points below are critical points of f. Then, if they are, classify them as local maxima, local minima, or saddle points.

(a) $P = (0,0)$.

(b) $Q = (0,2)$.

(c) $R = (4,0)$.

(d) $S = (4,2)$.

7. (**Extra credit: 10 points**)

Match the functions to the contour maps. (Obviously, some of the functions will not be matched.) [No justification is necessary on this problem.]

(a) $z = |x| + |y|$

(b) $z = \cos(x - y)$

(c) $z = \dfrac{1}{1 + x^2 + y^2}$

(d) $z = \cos(y^2)e^{(x^2+y^2)}$

(e) $z = x + y$

(f) $z = x^3 - y$

(g) $z = x^2 - y$