Math 2163

Jeff Mermin's section, Test 1, September 24

On the essay questions (# 2–8) write legibly in complete sentences, in such a way that I can easily tell what you are doing and why.

Put away your calculators, cell phones, smart watches, and other internet-enabled devices for the duration of the exam. If any of these are out for any reason, it will constitute an academic integrity violation.

6-week grade estimate

Category	**Score**	**Possible**
Exam 1		150
WebAssign		56
Written homework		20
Quizzes		37
Total		263

1. (**30 points**)Indicate whether the following statements are true or false. ("True" means "Always true", "false" means "sometimes false".) No justification is necessary on this problem. **Write the entire word "True" or "False"**. Illegible or abbreviated answers will receive no credit.

   In the statements below, a and b are scalars, $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are vectors, and x, y and z are the usual rectangular coordinates for $\mathbb{R}^3$.

   (a) The lines $\langle x, y, z\rangle = \langle -2, 3, 2\rangle + \langle 3, 4, -4\rangle\, t$ and $\langle x, y, z\rangle = \langle 0, -1, -4\rangle + \langle -5, 0, -3\rangle\, t$ intersect at the point $(-5, -1, 6)$.

   (b) The angle between two planes is equal to the angle between their normal vectors.

   (c) If ℓ is the line through $P = (1, 2, 3)$ and $Q = (4, 5, 6)$, then $\langle 4, 5, 6\rangle$ is a direction vector for ℓ.

   (d) Two lines in $\mathbb{R}^3$ define a plane.

   (e) $\mathbf{v} - \mathbf{w} = \mathbf{w} - \mathbf{v}$.

   (f) $\mathbf{v} \times \mathbf{w} = \mathbf{w} \times \mathbf{v}$.

   (g) $\mathbf{u} \bullet (\mathbf{v} + \mathbf{w}) = \mathbf{u} \bullet \mathbf{v} + \mathbf{u} \bullet \mathbf{w}$.

   (h) The vector $\langle 2, 3, 4\rangle$ is parallel to the plane with equation $2x+3y+4z = 5$.

   (i) If $\mathbf{v} \bullet \mathbf{w} > 0$, then $\mathbf{v}$ and $\mathbf{w}$ form an acute angle.

   (j) $a(b\mathbf{v}) = (ab)\mathbf{v}$.

2. (**25 points**) Let $\mathbf{v} = \langle 10, 3, -1 \rangle$ and $\mathbf{w} = \langle 9, -2, -5 \rangle$. Compute the following:

(a) $5\mathbf{v} + 2\mathbf{w}$

(b) $\mathbf{v} \bullet \mathbf{w}$

(c) $\mathbf{v} \times \mathbf{w}$

(d) $(\mathbf{v} \times \mathbf{w}) \bullet \mathbf{v}$

3. (**15 points**) Find the area of the parallelogram with vertices $(4, 2, 3)$, $(4, -1, 0)$, $(0, 6, -3)$, and $(0, 9, 0)$.

4. (**20 points**) Determine whether the line $\ell : (x, y, z) = (3, -1, 1) + t\langle 3, -3, -1\rangle$ and the plane $F : 2y + 2z = 5$ intersect. If they intersect, find the point of intersection. If they don't, find the distance from ℓ to F.

5. (**20 points**) Find the distance from the plane $G : 2x - 4y + 10z = 6$ to the point $P = (-3, 10, 9)$.

6. (**20 points**) A UFO is spotted at point $(1, 2, 3)$ at time $t = 0$. It is observed to have velocity $v(t) = \langle e^t, \sqrt{t}, \cos t \rangle$. Where is it at time $t = 3$?

7. (**20 points**) Find the length of the curve $r(t) = (\sin(\frac{\pi}{2}t), t^2, t^2 - 2t - 3)$ between the points $(1, 1, -4)$ and $(-1, 9, 0)$. Leave your answer as a definite integral (that a calculus 2 student would understand) and **do not evaluate**.

8. (**Extra credit: 20 points**) Our friend who goes to OU has encountered the problem "Find a parametrization of the tangent line to the curve $\mathbf{r}(t) = (t^3, t^6, t^9)$ at the origin.

His solution is $(x, y, z) = (0, 0, 0) + t\langle 0, 0, 0\rangle$.

(a) Without doing any work, explain why this cannot possibly be the correct answer.

(b) Our friend's work is as follows:

> The derivative is $(3t^2, 6t^5, 9t^8)$. We plug in $x = 0$ to get $3(0)^2 = 0$, $y = 0$ to get $6(0)^5 = 0$, and $z = 0$ to get $9(0)^8 = 0$. So the tangent vector is $\mathbf{v} = (0, 0, 0)$. The line is $P + t\mathbf{v} = (0, 0, 0) + t\langle 0, 00\rangle$.

What, if anything, has he done wrong? Explain.

(c) Identify a more useful parametrization for the curve and use it to find the correct tangent line. Briefly explain what's going on.