Math 2163

Jeff Mermin's section, Test 3, November 22

On the essay questions (# 2–4) write legibly in complete sentences, in such a way that I can easily tell what you are doing and why.

Put away your calculators, cell phones, smart watches, and other internet-enabled devices for the duration of the exam. If any of these are out for any reason, it will constitute an academic integrity violation.

14-week grade estimate

Category	**Score**	**Possible**
Exam 1		150
Exam 2		150
Exam 3		150
WebAssign		135
Written homework		40
Quizzes		100
Total		725

1. (**30 points**)Indicate whether the following statements are true or false. ("True" means "Always true", "false" means "sometimes false".) No justification is necessary on this problem. **Write the entire word "True" or "False"**. Illegible or abbreviated answers will receive no credit.

   In the statements below, a, b, h, and k are numbers, x and y are the usual rectangular coordinates, f is a smoothly differentiable function, $\mathbf{v}$ is a vector, $\mathbf{u}$ is a unit vector, D is a closed and bounded region, and $dA = dxdy$, as in the text.

   (a) $\nabla f(a,b)$ is tangent to the level curve of $f(x,y)$ passing through (a,b).

   (b) $\displaystyle\int_a^b \int_c^d f(x,y)dxdy = \int_a^b \int_c^d f(x,y)dydx.$

   (c) The angle between two planes is equal to the angle between their normal vectors.

   (d) If f has an absolute maximum on the region D at (a,b), then f has a local maximum at (a,b).

   (e) If $f(x,y) = x + y$, then $|D_{\mathbf{u}}(f)(x,y)| \leq 2$ for all x and y.

   (f) If $L(x,y) = f(a,b)+f_x(a,b)(x-a)+f_y(a,b)(y-b)$ is the linearization of f at (a,b) and $f_{xx} = f_{yy} = 0$, then $f(a+h,b+k) = L(a+h,b+k)$.

   (g) $\displaystyle\iint_R f\,dxdy = \iint_R f\,drd\theta.$

   (h) If $\mathbf{v}$ is a direction vector for a line ℓ, then $2\mathbf{v}$ is also a direction vector for ℓ.

   (i) $\mathbf{v}$ and $-2\mathbf{v}$ are parallel.

   (j) If $\displaystyle\iint_D f dA \geq \iint_D g dA$, then $f(x,y) \geq g(x,y)$ for all (x,y) in D.

2. (**30 points**) Consider the double integral $I = \int_{y=0}^{y=1} \int_{x=\arcsin y}^{x=\frac{\pi}{2}} \cos x \sqrt{1+\cos^2 x}\, dx dy.$

   (a) Rewrite I with the x-integral on the outside and the y-integral on the inside.

   (b) Evaluate I, in whichever form makes more sense. [You should not need the integral tables for this, but they're attached at the back of the exam.]

3. (**90 points**) Express each of the following as an iterated integral. You may use any coordinate system that makes sense, provided that you define anything other than rectangular, polar, or spherical coordinates. **Do not evaluate your integrals.**

   (a) $\iiint_R 2x\,dxdydz$, where R is the region in the first octant bounded by the curves $x = \sqrt{4 - y^2}$ and $y = z$.

   (b) The volume of the region inside both the cylinder $(x-1)^2 + y^2 = 1$ and the sphere $x^2 + y^2 + z^2 = 1$.

(c) $\displaystyle\iint_R (4x+8y)\,dy\,dx$, where R is the parallelogram with vertices $(-1,3)$, $(1,-3)$, $(3,-1)$, and $(1,5)$.

4. (**Extra credit: 20 points**) Suppose w, x, y, and z are randomly chosen numbers between 0 and 1 (with a uniform distribution). Compute the following probabilities. Explain your work. [You may express your answers as numbers or as iterated integrals.]

   (a) $\mathrm{P}(w < x \text{ and } y < z)$.

   (b) $\mathrm{P}(w < x < y < z)$.

(c) $\mathrm{P}(w + x + y < z)$.

(d) $\mathrm{P}(w < x \text{ and } w + x < y \text{ and } w + x + y < z)$.

(e) $\mathrm{P}(w^2 + x^2 + y^2 + z^2 < 1)$.