Math 2163

Jeff Mermin's section, Test 2, October 11

On the essay questions (# 2–8) write legibly in complete sentences, in such a way that I can easily tell what you are doing and why.

Put away your calculators, cell phones, smart watches, and other internet-enabled devices for the duration of the exam. If any of these are out for any reason, it will constitute an academic integrity violation.

10-week grade estimate

Category	**Score**	**Possible**
Exam 1		150
Exam 2		150
WebAssign		75
Written homework		20
Quizzes		62
Total		457

1. (**30 points**)Indicate whether the following statements are true or false. ("True" means "Always true", "false" means "sometimes false".) No justification is necessary on this problem. **Write the entire word "True" or "False"**. Illegible or abbreviated answers will receive no credit.

   In the statements below, x, y, and z are the usual rectangular coordinates for $\mathbb{R}^3$, $f = f(x, y)$ and $g = g(x, y)$ are smoothly differentiable functions, s and t are parameters, $\mathbf{r} = \mathbf{r}(t) = (x(t), y(t), z(t))$ is a smooth curve, P is a point, and $\mathbf{v}$ is a vector.

   (a) $\dfrac{d\mathbf{r}}{dt} = \left\langle \dfrac{dx}{dt}, \dfrac{dy}{dt}, \dfrac{dz}{dt} \right\rangle$.

   (b) There is a function $g(x, y)$ such that $g_x(x, y) = x^2 + y^2$ and $g_y(x, y) = x^2 - y^2$.

   (c) The projection of $\mathbf{v}$ along itself, $\text{proj}_{\mathbf{v}}(\mathbf{v})$, is equal to $\mathbf{v}$.

   (d) $\displaystyle\int_{t=0}^{t=1} \frac{d\mathbf{r}}{dt} dt = \mathbf{r}(1) - \mathbf{r}(0)$.

   (e) The planes $x + y + z = 1$ and $2x + 2y + 2z = 1$ are parallel.

   (f) If $z = f(x, y)$ and x and y are both functions of two independent variables s and t, $x = x(s, t)$ and $y = y(s, t)$, then $z_t = z_x x_t + z_y y_t$.

   (g) The equations $(x, y, z) = P + t\mathbf{v}$ and $(x, y, z) = P - t\mathbf{v}$ define the same line.

   (h) If f has two local maxima, then it must have a local minimum.

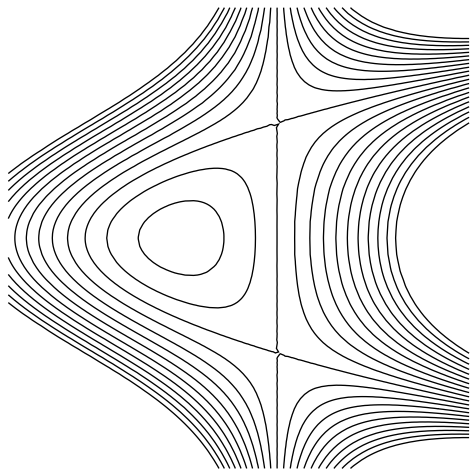
   (i) $\nabla(f + g) = \nabla f + \nabla g$.

   (j) $\dfrac{\partial}{\partial x}(fg) = \left(\dfrac{\partial f}{\partial x}\right)\left(\dfrac{\partial g}{\partial x}\right)$.

2. (**30 points**) Compute all four of the second partial derivatives for

$$f(x, y) = (x - y)(x + y)(x^2 - y^2).$$

3. (**25 points**) Consider the function $z = f(x, y)$ whose contour map is given.



(a) Does this function have any critical points? If so, mark them in the graph and identify each point as either a saddle point or a local optimum. (The contour lines aren't labeled, so we can't distinguish between maxima and minima.)

(b) Which of the following functions could have the given contour map? Explain briefly.

(a) $z = \cos x - \sin y$

(b) $z = e^{xy}$

(c) $z = \cos(x - y)$

(d) $z = x(x - y^2 + 1)$

4. (**20 points**) Find an equation for the tangent plane to the function

$$z = \frac{4y^2}{x^2+1}$$

at the point $(3, 2, 1.6)$.

5. (**20 points**) Find an equation for the tangent plane to the surface

$$xy^2 - y \ln z = 12$$

at the point $(3, 2, 1)$.

6. (**25 points**) Consider the function $f(x,y) = x^3 + 9xy - 9y^2 - 21x$. You may assume that

$$f_x = 3x^2 + 9y - 21 \qquad\qquad f_y = 9x - 18y$$

$$f_{xx} = 6x \qquad f_{xy} = 9 \qquad f_{yy} = -18$$

Decide whether the points below are critical points of f. Then, if they are, classify them as local maxima, local minima, or saddle points.

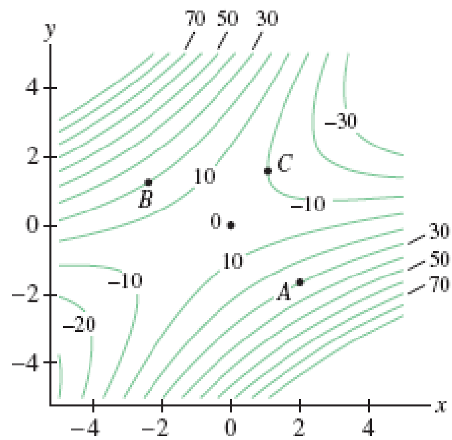
(a) $P = (0,0)$.

(b) $Q = (1,2)$.

(c) $R = \left(-\frac{7}{2}, -\frac{7}{4}\right)$.

(d) $S = (2,1)$.

7. (**Extra credit: 10 points**) One of the WebAssign problems from section 14.3 asked us for information about the partial derivatives of a function based on its contour map, reproduced below.



Contour map of $f(x, y)$.

Explain why this cannot be the contour map of a continuous function.

8. (**Extra credit: 10 points**) Give examples of functions with the following properties, and verify that your examples are correct.

(a) $f(x, y)$ such that $\lim_{x\to 0} f(x, 0) = \lim_{y\to 0} f(0, y) = 0$ but $\lim_{(x,y)\to(0,0)} f(x, y)$ does not exist.

(b) $f(x, y)$ such that $\lim_{x\to 0} f(x, mx) = 0$ for every possible slope m but $\lim_{(x,y)\to(0,0)} f(x, y)$ does not exist.