Math 2163

Jeff Mermin's section, Test 1, September 20

On the essay questions (# 2–8) write legibly in complete sentences, in such a way that I can easily tell what you are doing and why.

Put away your calculators, cell phones, smart watches, and other internet-enabled devices for the duration of the exam. If any of these are out for any reason, it will constitute an academic integrity violation.

6-week grade estimate

Category	Score	Possible
Exam 1		150
WebAssign		45
Written homework		20
Quizzes		37
Total		252

1. (**30 points**)Indicate whether the following statements are true or false. ("True" means "Always true", "false" means "sometimes false".) No justification is necessary on this problem. **Write the entire word "True" or "False"**. Illegible or abbreviated answers will receive no credit.

   In the statements below, a, b, c, d, and d' are numbers, $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are vectors in $\mathbb{R}^3$, x, y, and z are the usual rectangular coordinates for $\mathbb{R}^3$,

   (a) If ℓ is the line through $P = (1, 2, 3)$ and $Q = (4, 5, 6)$, then $\langle 4, 5, 6\rangle$ is a direction vector for ℓ.

   (b) The planes $x + y + z = 1$ and $2x + 2y + 2z = 1$ are parallel.

   (c) $(\mathbf{u} \bullet \mathbf{v})\mathbf{w} = \mathbf{u}(\mathbf{v} \bullet \mathbf{w})$.

   (d) $\mathbf{v}$ and $-2\mathbf{v}$ point in the same direction.

   (e) The distance between the planes $F : ax + by + cz = d$ and $G : ax + by + cz = d'$ is $|d - d'|$.

   (f) If $\mathbf{v} \bullet \mathbf{w} < 0$, then $\mathbf{v}$ and $\mathbf{w}$ form an acute angle.

   (g) If $\mathbf{v} \bullet \mathbf{w} > 0$, then $\mathbf{v}$ and $\mathbf{w}$ form an acute angle.

   (h) $\mathbf{u} \times (\mathbf{v} + \mathbf{w}) = \mathbf{u} \times \mathbf{v} + \mathbf{u} \times \mathbf{w}$.

   (i) $a(b\mathbf{v}) = (ab)\mathbf{v}$.

   (j) $\mathbf{v}$ and $-2\mathbf{v}$ are parallel.

2. (**20 points**) Let $\mathbf{v} = \langle 1, -3, 0 \rangle$ and $\mathbf{w} = \langle 0, -2, 1 \rangle$. Compute the following:

   (a) $\mathbf{w} - 2\mathbf{v}$.

   (b) $\mathbf{v} \bullet \mathbf{w}$.

   (c) $\mathbf{v} \times \mathbf{w}$.

   (d) $(\mathbf{v} + \mathbf{w}) \times \mathbf{w}$.

3. (**10 points**) Find two points on the plane $2x - y + z = 1$.

4. (**20 points**) Verify that the two lines $\ell : (x, y, z) = (3, 1, 1) + \langle 1, -2, 4\rangle t$ and $m : (x, y, z) = (2, -3, 1) + \langle 1, 1, 2\rangle t$ intersect. Then find the equation of the plane containing them both.

5. (**20 points**) Find an equation of the plane containing the parallel lines $\ell : (-2, 0, 5) + \langle 0, -1, 4\rangle t$ and $m : (-3, 0, -1) + \langle 0, -1, 4\rangle t$.

6. (**20 points**) Find a parametrization of the normal line to the plane $F : x + 2y - 3z = 1$ that passes through the point $(5, 4, -1)$.

7. (**40 points**) Consider the curve $C : \mathbf{r}(t) = (\sin t, \sin 2t, \cos t)$

(a) C forms a loop in $\mathbb{R}^3$. Express the length of this loop as an integral. **Leave your answer as an integral that a calculus II student would understand. Do not evalulate.**

(b) Find a parametrization of the tangent line to C at the point $\left(\frac{\sqrt{2}}{2}, 1, \frac{\sqrt{2}}{2}\right)$.

8. (**Extra credit: 20 points**) The ten parametric equations below include equations for the five curves below. Match each curve to its equation. (No justification is necessary for correct answers here, but good reasoning leading to wrong answers may earn partial credit.

(a) $(t^2, t^2 - 2t - 3, t - 3)$

(b) $(t, t^3, t^2 + 1)$

(c) $(t, t^2, t + 1)$

(d) $(t, t, \dfrac{25t}{1 + t^2})$

(e) $(\cos^3 t, \sin^3 t, \sin 2t)$

(f) $(7, 12 \cos t, 12 \sin t)$

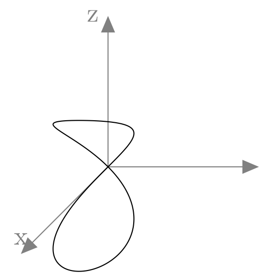
(g) $(\sin t, 0, 4 + \cos t)$

(h) $(t \cos t, t \sin t, t)$

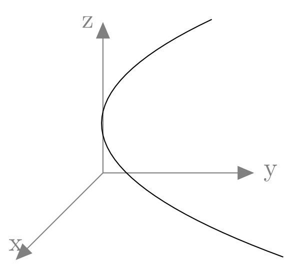
(i) $(\sin t, \cos t, \sin t \cos 2t)$

(j) $(\sin^2 t, \sin t \;\cos t, \sin t)$

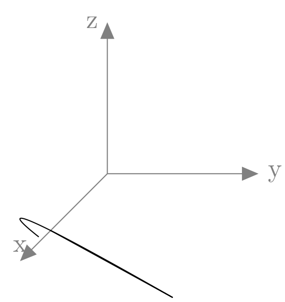
(I)



(II)



(III)



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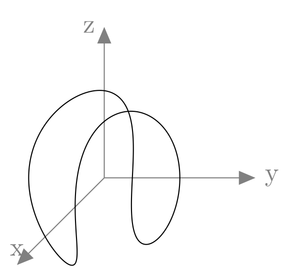

(V)