Math 2163

Jeff Mermin's section, Test 3, November 17

On the essay questions (# 2–6) write legibly in complete sentences, in such a way that I can easily tell what you are doing and why.

You won't need the integral tables on the following page, but they're available anyway.

14-week grade estimate

Category	**Score**	**Possible**
WebAssign		123
Exam 1		150
Exam 2		150
Exam 3		150
hline Quizzes		100
Extra Credit		0
Total		673

1. (**30 points**)Indicate whether the following statements are true or false. ("True" means "Always true", "false" means "sometimes false".) No justification is necessary on this problem. **Write the entire word "True" or "False"**. Illegible or abbreviated answers will receive no credit.

   In the statements below, a, b, and c are numbers, D is a region in $\mathbb{R}^2$, f is a smoothly differentiable function on $\mathbb{R}^2$, L is the linearization of f at the point (a, b), F is a smoothly differentiable function on $\mathbb{R}^3$, $\mathbf{v}$ and $\mathbf{w}$ are vectors in $\mathbb{R}^3$, x, y, and z are the usual rectangular coordinates for $\mathbb{R}^3$, ρ, θ, and ϕ are the usual spherical coordinates, and u and v are some other coordinates for $\mathbb{R}^2$.

   (a) $a(b\mathbf{v}) = (ab)\mathbf{v}$.

   (b) The tangent plane to the graph of the function $z = f(x, y)$ at the point $(a, b, f(a, b))$ is the graph of the linearization of f at (a, b).

   (c) $\left|\begin{array}{cc} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{array}\right| \left|\begin{array}{cc} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{array}\right| = 1.$

   (d) $\nabla F(a, b, c)$ is normal to the surface $F(x, y, z) = 0$ at the point (a, b, c).

   (e) If $f_{xx} = f_{yy} = 0$, then $f(a + h, b + k) = L(a + h, b + k)$.

   (f) $\mathbf{v}$ and $\mathbf{w}$ are perpendicular if and only if $\mathbf{v} \times \mathbf{w} = \mathbf{0}$.

   (g) The area of D is $\displaystyle\iint_D 1 dA$.

   (h) If D is neither closed nor bounded, then f fails to have a global maximum on D.

   (i) If $\mathbf{v}$ is a direction vector for a line ℓ, then $2\mathbf{v}$ is also a direction vector for ℓ.

   (j) If S is the sphere of radius one about the origin, then $\displaystyle\iiint_S f dV = \int_{\phi=0}^{\phi=2\pi} \int_{\theta=0}^{\theta=2\pi} \int_{\rho=0}^{\rho=1} f\rho^2 \sin\phi \, d\rho d\theta d\phi$.

2. (**20 points**) Evaluate $\displaystyle\int_{x=-\frac{\pi}{3}}^{x=\frac{\pi}{3}} \int_{y=0}^{y=\sec x} \cos x \, dydx$.

3. (**20 points**) The integral $\displaystyle\int_{x=0}^{x=1} \int_{y=0}^{y=1} \arctan(xy) \, dydx$ cannot be evaluated algebraically. Estimate its value using a Riemann sum with at least six summands. **Leave your answer as a Riemann sum**, and explain your work clearly enough that a reader could tell where everything is coming from.

4. (**40 points**) Let $I = \displaystyle\int_{y=0}^{y=9} \int_{x=\sqrt{9}}^{x=3} \frac{x}{\sqrt{x^2+y}}\, dxdy.$

(a) Reverse the order of integration.

(b) Evaluate I, using whichever order seems most convenient.

5. (**40 points**) Express **two** of the following three quantities as an iterated integral in an appropriate coordinate system. If you use coordinates other than rectangular, polar (cylindrical), or spherical, you must clearly define them. **Do not evaluate the integrals**. Instead, leave your answers as iterated integrals. **Make it very clear which two problems you are solving**, for example by circling their letters or x-ing out the space (and any work) allotted to the third. I will not grade all three.

   (a) The volume of the region in the first octant bounded by the surfaces $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$.

(b) $\displaystyle\iint_W (x+2y)e^{y-x}\,dydx$, where W is the parallelogram with vertices $(0,0)$, $(2,2)$, $(0,3)$, and $(-2,1)$. [There is a way to express this integral using one iterated integral. There is also a significantly easier way to express it as a sum of two iterated integrals. If your answer is a sum of two iterated integrals, you will earn no more than half credit.]

(c) The mass of the wedge bounded by the curves $x^2+y^2=4$, $z=0$, and $z-y=2$, if its density at the point (x,y,z) is equal to x^2y^2z.

6. (**Extra credit: 20 points**) Our friend the OU student has been learning how to set up integrals. He works four problems and produces the answers below. In each case, we have good reason to believe, even without seeing the questions, that the answers are wrong. Can we say for sure that they're wrong, or are we merely skeptical? Identify all likely issues with these integrals, and explain why they're causes for concern.

- $\displaystyle\int_{\theta=0}^{\theta=6\pi} \int_{r=\theta}^{r=\theta+\pi} \theta dr d\theta.$

- $\displaystyle\int_{x=0}^{x=1} \int_{z=y^2}^{z=y} \int_{y=x}^{y=2x} (x^2+y^2+z^2) dy dz dx.$

- $\displaystyle\int_{\rho=-2}^{\rho=2} \int_{\theta=0}^{\theta=\pi} \int_{\phi=\frac{\pi}{2}}^{\phi=\frac{3\pi}{2}} \rho^2 \cos\phi d\phi d\theta d\rho.$

- $\displaystyle\int_{x=0}^{x=1} \int_{y=x}^{y=x^2} (y^2-x^2) dy dx.$