 Name:___________________________

Math 2163

Jeff Mermin's section, Test 2, October 27

On the essay questions (# 3–8) write legibly in complete sentences, in such a way that I can easily tell what you are doing and why.

Do not evaluate any integrals on this test. If you would take an integral, instead simplify the integrand and the limits of integration (if any), and leave the integral as your final answer.

10-week grade estimate

Category	**Score**	**Possible**
Exam 1		150
Exam 2		150
WebAssign		102
Quizzes		57
Total		459

1. (**30 points**)Indicate whether the following statements are true or false. ("True" means "Always true", "false" means "sometimes false".) No justification is necessary on this problem. **Write the entire word "True" or "False"**. Illegible or abbreviated answers will receive no credit.

   In the statements below, a, b, c, L, p, q, and r are numbers, $\mathbf{a}$ and $\mathbf{b}$ are vectors in $\mathbb{R}^3$, P is a point in $\mathbb{R}^3$, t is a parameter, x, y, and z are the usual rectangular coordinates for $\mathbb{R}^3$, $f = f(x, y)$ is a smoothly differentiable function defined on $\mathbb{R}^2$, and $F = F(x, y, z)$ is a smoothly differentiable function defined on $\mathbb{R}^3$.

   (a) If $\lim_{x\to 0} f(x, 0) = \lim_{y\to 0} f(0, y) = L$, then $\lim_{(x,y)\to(0,0)} f(x, y) = L$.

   (b) The point $P = (p, q, r)$ is contained in the plane with equation $a(x - p) + b(y - q) + c(z - r) = 0$.

   (c) If $\mathbf{u}$ is the unit vector in the direction of ∇F at P, then $D_{\mathbf{u}}(F)(P) = \|\nabla F\|$.

   (d) If the differentiable function f has a local maximum at $(0, 0)$, then $f_x(0, 0) = 0$.

   (e) There is a function $g(x, y)$ such that $g_x(x, y) = x^2 + y^2$ and $g_y(x, y) = x^2 - y^2$.

   (f) If f has two local maxima, then it must have a local minimum.

   (g) $\nabla F(p, q, r)$ is normal to the surface $F(x, y, z) = 0$ at the point (p, q, r).

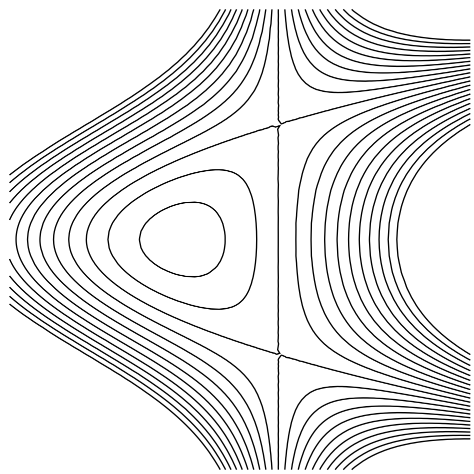
   (h) The lines $(x, y, z) = (-2, 3, 2) + \langle 3, 4, -4\rangle\, t$ and $(x, y, z) = (0, -1, -4) + \langle -5, 0, -3\rangle\, t$ intersect at the point $(-5, -1, 6)$.

   (i) $|\mathbf{a} \bullet \mathbf{b}| = \|\mathbf{a}\|\|\mathbf{b}\|$.

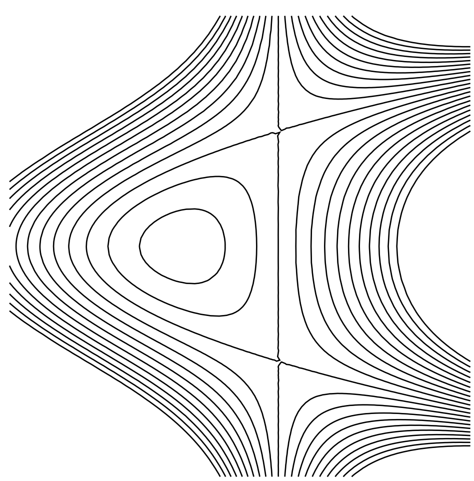
   (j) If $\mathbf{a} \bullet \mathbf{b} < 0$, then $\mathbf{a}$ and $\mathbf{b}$ form an acute angle.

2. (**20 points**) Consider the function $z = f(x, y)$ whose contour map is given. Assume that f has no local minima.

   (a) Does this function have any critical points? If so, mark them in the graph and identify each point as either a saddle point or a local maximum. (Since we assume no local minima, any local extremum must be a maximum.)



   (b) Another copy of the contour map is provided below. Assume that the graph is for the region $-2 \leq x \leq 2$ and $-2 \leq y \leq 2$. Draw in copies of the gradient vector ∇f at $(0, 0)$ and $(0, 1)$. Then state which of the two gradient vectors is longer, and explain how you know.



3. (**30 points**) Let $f(x,y) = (x^2 + y^2)(x+y)(x-y)$. Find all four second partial derivatives of f.

4. (**20 points**) Let S be the surface defined by the equation $z = \cos(xy^2)$. Find an equation for the tangent plane to S at $\left(\pi, \frac{1}{2}, \frac{1}{\sqrt{2}}\right)$.

5. (**20 points**) Let T be the surface defined by the equation $xe^{yz} + x^2y = 3$. Find an equation for the tangent plane to T at $(1, 2, 0)$.

6. (**30 points**) Let $f(x, y) = 4x - 3x^3 - 2xy^2$. You may assume that

$$f_x = 4 - 9x^2 - 2y^2 \qquad\qquad f_y = -4xy$$

$$f_{xx} = -18x \qquad f_{xy} = -4y \qquad f_{yy} = -4x$$

Decide whether the points below are critical points of f. Then, if they are, classify them as local maxima, local minima, or saddle points.

(a) $P = (0, 0)$

(b) $Q = \left(\frac{2}{3}, 0\right)$

(c) $R = \left(0, \sqrt{2}\right)$

(d) $R = \left(\sqrt{2}, \frac{2}{3}\right)$

7. (**Extra credit: 10 points**) An OU student is learning about limits of multivariable functions, and asserts the following:

$$\text{If } \lim_{x\to 0} f(x,0) = \lim_{y\to 0} f(0,y), \text{ then } \lim_{(x,y)\to(0,0)} f(x,y) \text{ exists.}$$

Prove him wrong by providing an example of a function where the first two limits are equal but the third limit doesn't exist. Then explain his error in as much detail as possible.

8. (**Extra credit: 10 points**) A scientist wants to measure a quantity X, but can only do so indirectly. She instructs her graduate students measure related quantities w, y, and z, then uses the formula $X = w^2 - 3yz + 2wz$ to compute X. The graduate students produce measurements $w = 20 \pm 1.12$, $y = 6 \pm .04$, and $z = 100 \pm .9$. This allows the scientist to compute $X = 2600 \pm E_{\text{RROR}}$. What is the value of E_{RROR}?