Math 2163

Jeff Mermin's section, Test 1, September 22

On the essay questions (# 2–10) write legibly in complete sentences, in such a way that I can easily tell what you are doing and why.

1. (**15 points**)Indicate whether the following statements are true or false. ("True" means "Always true", "false" means "sometimes false".) No justification is necessary on this problem. **Write the entire word "True" or "False"**. Illegible or abbreviated answers will receive no credit.

   In the statements below, P and Q are points, $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are vectors in $\mathbb{R}^3$, t is a parameter, x, y, and z are variables which are each differentiable functions of t, and $\mathbf{r} = \mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$ is a parametrized curve.

   (a) If ℓ is the line through $P = (1, 2, 3)$ and $Q = (4, 5, 6)$, then $\langle 4, 5, 6 \rangle$ is a direction vector for ℓ.

   (b) $(\mathbf{u} \bullet \mathbf{v})\mathbf{w} = \mathbf{u}(\mathbf{v} \bullet \mathbf{w})$.

   (c) $\dfrac{d}{dt} \|\mathbf{r}\| = \left\| \dfrac{d\mathbf{r}}{dt} \right\|$.

   (d) The equations $(x, y, z) = P + t\mathbf{v}$ and $(x, y, z) = P - t\mathbf{v}$ define the same line.

   (e) $\mathbf{v}$ and $\mathbf{w}$ are parallel if and only if $\mathbf{v} \bullet \mathbf{w} = \mathbf{0}$.

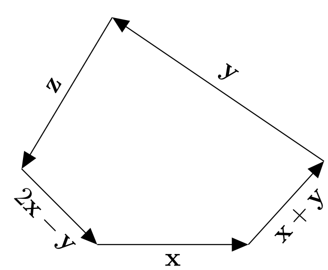
2. (**20 points**) Let $\mathbf{x} = \langle 3, 1, 1 \rangle$ and $\mathbf{y} = \langle -5, 8, -3 \rangle$. Compute the following:

   (a) $2\mathbf{x} - 3\mathbf{y}$.

   (b) $\mathbf{x} \bullet \mathbf{y}$.

   (c) $\mathbf{x} \times \mathbf{y}$.

   (d) $\mathbf{z}$, based on this diagram (not to scale):



3. (**15 points**) Find two points on the line $(x, y, z) = (-1, 4, 5) + \langle -4, 4, 3 \rangle t$.

4. (**20 points**) Verify that the two lines $\ell : (x, y, z) = (8, 2, 8) + \langle 10, 2, -2 \rangle t$ and $m : (x, y, z) = (-3, -9, -1) + \langle 4, 3, 2 \rangle t$ intersect. Then find the equation of the plane containing them both.

5. (**20 points**) Find the distance from the point $(2, -3, -2)$ to the line $(x, y, z) = (-1, 2, 0) + (1, -1, 4)t$.

6. (**20 points**) Find a parametrization of the tangent line to $\mathbf{r}(t) = \left(t^2, t^3, t^4\right)$ at the point $P = (4, 8, 16)$.

7. (**20 points**) Find the length of the curve $\mathbf{r}(t) = \left(t^2, t+1, e^{-t}\right)$ between points $P = (1, 0, e)$ and $Q = (0, 1, 1)$. **Leave your answer as an integral that a calculus II student would understand. Do not evaluate.**

8. (**10 points**) An object moves through a force field with acceleration vector $\mathbf{a}(t) = \langle e^t, 2t, 1\rangle$. At time $t = 0$, it passes through the point $(2, 1, 1)$ with velocity $\langle 1, 0, 1\rangle$. Where will it be at time $t = 4$?

9. (**Extra credit: 20 points**) The ten parametric equations below include equations for the five curves below. Match each curve to its equation. (No justification is necessary for correct answers here, but good reasoning leading to wrong answers may earn partial credit.

(a) $(t^2, t^2 - 2t - 3, t - 3)$

(b) $(t, t^3, t^2 + 1)$

(c) $(t, t^2, t + 1)$

(d) $(t, t, \dfrac{25t}{1+t^2})$

(e) $(\cos^3 t, \sin^3 t, \sin 2t)$

(f) $(7, 12\cos t, 12\sin t)$

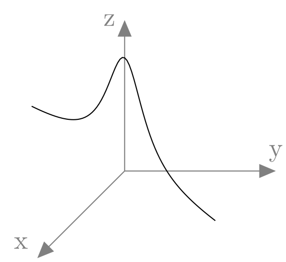
(g) $(\sin t, 0, 4 + \cos t)$

(h) $(t\cos t, t\sin t, t)$

(i) $(\sin t, \cos t, \sin t \cos 2t)$

(j) $(\sin^2 t, \sin t\ \cos t, \sin t)$

(I)



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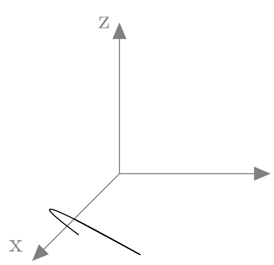

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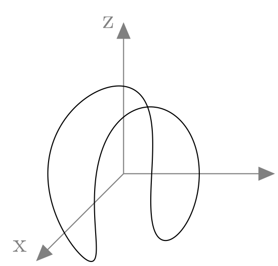

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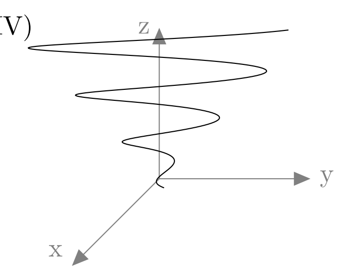

(V)