Math 2163

Jeff Mermin's section, Test 2, October 21

On the essay questions (# 3–8) write legibly in complete sentences, in such a way that I can easily tell what you are doing and why.

Do not evaluate any integrals on this test. If you would take an integral, instead simplify the integrand and the limits of integration (if any), and leave the integral as your final answer.

1. (**30 points**)Indicate whether the following statements are true or false. ("True" means "Always true", "false" means "sometimes false".) No justification is necessary on this problem. **Write the entire word "True" or "False"**. Illegible or abbreviated answers will receive no credit.

   In the statements below, a, b, and L are numbers, $\mathbf{v}$ and $\mathbf{w}$ are vectors in $\mathbb{R}^3$, x, y, and z are the usual rectangular coordinates for $\mathbb{R}^3$, $f = f(x, y)$ and $g = g(x, y)$ are smoothly differentiable functions defined on $\mathbb{R}^2$, and D is a closed and bounded region inside $\mathbb{R}^2$.

   (a) If $\mathbf{v}$ is a direction vector for a line ℓ, then $2\mathbf{v}$ is also a direction vector for ℓ.

   (b) $(a + b)\mathbf{v} = a\mathbf{v} + b\mathbf{v}$.

   (c) $a(\mathbf{v} + \mathbf{w}) = a\mathbf{v} + a\mathbf{w}$.

   (d) If $\mathbf{v} \bullet \mathbf{w} < 0$, then $\mathbf{v}$ and $\mathbf{w}$ form an acute angle.

   (e) If $\lim_{(x,y)\to(a,b)} f(x, y) = L$, then $\lim_{(x,y)\to(a,b)} (\cos f(x, y)) = \cos L$.

   (f) $\mathbf{v}$ and $\mathbf{w}$ are parallel if and only if $\mathbf{v} \times \mathbf{w} = \mathbf{0}$.

   (g) There are functions $h_1(x)$ and $h_2(y)$ such that $f(x, y) = h_1(x)+h_2(y)$.

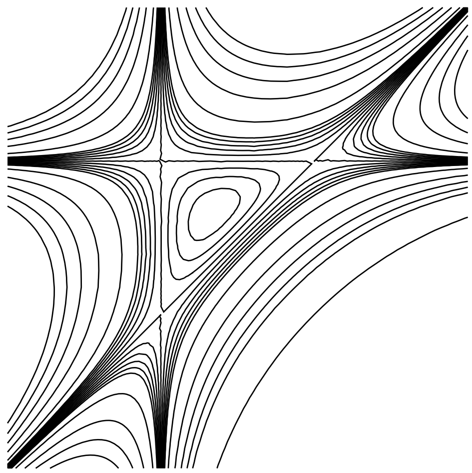
   (h) $\nabla(f + g) = \nabla f + \nabla g$.

   (i) If f has a local maximum at $(0, 0)$, then $f_x(0, 0) = 0$.

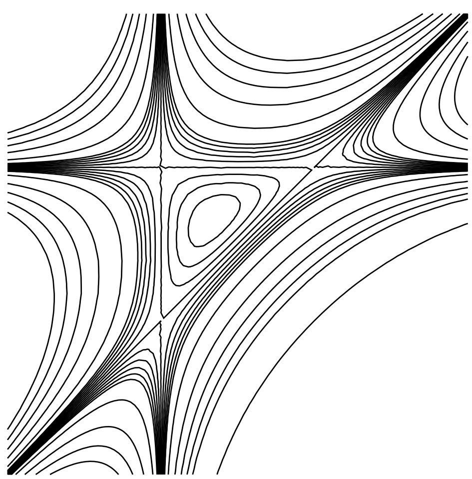
   (j) If f has an absolute maximum on the region D at (a, b), then f has a local maximum at (a, b).

2. (**20 points**) Consider the function $z = f(x, y)$ whose contour map is given.

   (a) Does this function have any critical points? If so, mark them in the graph and identify each point as either a saddle point or a local extremum. (We don't have enough information to determine if we're looking at maxima or minima.)



   (b) Another copy of the contour map is provided below. Assume that the graph is for the region $-2 \le x \le 2$ and $-2 \le y \le 2$. Draw in the circle $x^2 + y^2 = 1$, and identify all points where z could achieve an absolute maximum or minimum on this circle. Write a sentence justifying your choice of these points in terms of ∇z and/or the slopes of the contour lines.



3. (**30 points**) Let $f(x,y) = \dfrac{(x+y)(x-y)}{xy}$. Find all four second partial derivatives of f.

4. (**20 points**) Let S be the surface defined by the equation

$$z = x^2 + 2y^2 - 12xy + y$$

Find the tangent plane to S at $(2, -1, 29)$.

5. (**15 points**) Let $f(x, y, z) = xe^{yz}$, and put $P = (1, 0, 2)$ and $\mathbf{v} = \langle 2, -1, 2 \rangle$. Find the unit vector $\mathbf{u}$ in the direction of $\mathbf{v}$, and the directional derivative $D_{\mathbf{u}}(f)(P)$.

6. (**20 points**) Let $f(x, y) = xy^2 - x^2y + xy$. You may assume that

$$f_x = y^2 - 2xy + y \qquad f_y = 2xy - x^2 + x$$

$$f_{xx} = -2y \qquad f_{xy} = 2y - 2x + 1 \qquad f_{yy} = 2x$$

Decide whether the points below are critical points of f. Then, if they are, classify them as local maxima, local minima, or saddle points.

(a) $P = (0, 0)$

(b) $Q = (0, 1)$

(c) $R = \left(\frac{1}{3}, \frac{1}{3}\right)$

(d) $R = \left(\frac{1}{3}, -\frac{1}{3}\right)$

7. (**15 points**) Use a technique from this course to estimate the value of

$$(15.02)(1.98)^2.$$

[Maybe you know some other techniques for efficient multiplication. If so, humor me. I could have asked a harder one.]

8. (**Extra credit: 10 points**)

Prove that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3y}{x^6+2y^2}$ does not exist, or provide strong evidence that it does.

9. (**Extra credit: 10 points**)

Match the functions to the contour maps. [No justification is necessary on this problem.]

(a) $(z = 2x - 3y)^2$

(b) $z = x^2y^2$

(c) $z = e^{-x}$

(d) $z = \sin^2 x + \cos^2 y$

(e) $z = \dfrac{1}{(x-1)(y+2)^2}$