Math 2163

Jeff Mermin's section, Final exam, December 11

On the essay questions (# 2–12) write legibly in complete sentences, in such a way that I can easily tell what you are doing and why.

You may or may not find the following tables helpful.

Basic Forms

1. $\int u^n\,du = \frac{u^{n+1}}{n+1} + C,\quad n \neq -1$
2. $\int \frac{du}{u} = \ln|u| + C$
3. $\int e^u\,du = e^u + C$
4. $\int a^u\,du = \frac{a^u}{\ln a} + C$
5. $\int \sin u\,du = -\cos u + C$
6. $\int \cos u\,du = \sin u + C$
7. $\int \sec^2 u\,du = \tan u + C$
8. $\int \csc^2 u\,du = -\cot u + C$
9. $\int \sec u \tan u\,du = \sec u + C$
10. $\int \csc u \cot u\,du = -\csc u + C$
11. $\int \tan u\,du = \ln|\sec u| + C$
12. $\int \cot u\,du = \ln|\sin u| + C$
13. $\int \sec u\,du = \ln|\sec u + \tan u| + C$
14. $\int \csc u\,du = \ln|\csc u - \cot u| + C$
15. $\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\frac{u}{a} + C$
16. $\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$

Exponential and Logarithmic Forms

17. $\int ue^{au}\,du = \frac{1}{a^2}(au-1)e^{au} + C$
18. $\int u^n e^{au}\,du = \frac{1}{a}u^n e^{au} - \frac{n}{a}\int u^{n-1}e^{au}\,du$
19. $\int e^{au}\sin bu\,du = \frac{e^{au}}{a^2+b^2}(a\sin bu - b\cos bu) + C$
20. $\int e^{au}\cos bu\,du = \frac{e^{au}}{a^2+b^2}(a\cos bu + b\sin bu) + C$
21. $\int \ln u\,du = u\ln u - u + C$
22. $\int u^n \ln u\,du = \frac{u^{n+1}}{(n+1)^2}[(n+1)\ln u - 1] + C$
23. $\int \frac{1}{u\ln u}\,du = \ln|\ln u| + C$

Hyperbolic Forms

24. $\int \sinh u\,du = \cosh u + C$
25. $\int \cosh u\,du = \sinh u + C$
26. $\int \tanh u\,du = \ln\cosh u + C$
27. $\int \coth u\,du = \ln|\sinh u| + C$
28. $\int \operatorname{sech} u\,du = \tan^{-1}|\sinh u| + C$
29. $\int \operatorname{csch} u\,du = \ln\left|\tanh\frac{1}{2}u\right| + C$
30. $\int \operatorname{sech}^2 u\,du = \tanh u + C$
31. $\int \operatorname{csch}^2 u\,du = -\coth u + C$
32. $\int \operatorname{sech} u \tanh u\,du = -\operatorname{sech} u + C$
33. $\int \operatorname{csch} u \coth u\,du = -\operatorname{csch} u + C$

Trigonometric Forms

34. $\int \sin^2 u\,du = \frac{1}{2}u - \frac{1}{4}\sin 2u + C$
35. $\int \cos^2 u\,du = \frac{1}{2}u + \frac{1}{4}\sin 2u + C$
36. $\int \tan^2 u\,du = \tan u - u + C$
37. $\int \cot^2 u\,du = -\cot u - u + C$
38. $\int \sin^3 u\,du = -\frac{1}{3}(2+\sin^2 u)\cos u + C$
39. $\int \cos^3 u\,du = \frac{1}{3}(2+\cos^2 u)\sin u + C$
40. $\int \tan^3 u\,du = \frac{1}{2}\tan^2 u + \ln|\cos u| + C$
41. $\int \cot^3 u\,du = -\frac{1}{2}\cot^2 u - \ln|\sin u| + C$
42. $\int \sec^3 u\,du = \frac{1}{2}\sec u\tan u + \frac{1}{2}\ln|\sec u + \tan u| + ($
43. $\int \csc^3 u\,du = -\frac{1}{2}\csc u\cot u + \frac{1}{2}\ln|\csc u - \cot u| + C$
44. $\int \sin^n u\,du = -\frac{1}{n}\sin^{n-1}u\cos u + \frac{n-1}{n}\int \sin^{n-2}u\,du$
45. $\int \cos^n u\,du = \frac{1}{n}\cos^{n-1}u\sin u + \frac{n-1}{n}\int \cos^{n-2}u\,du$
46. $\int \tan^n u\,du = \frac{1}{n-1}\tan^{n-1}u - \int \tan^{n-2}u\,du$
47. $\int \cot^n u\,du = \frac{-1}{n-1}\cot^{n-1}u - \int \cot^{n-2}u\,du$
48. $\int \sec^n u\,du = \frac{1}{n-1}\tan u\sec^{n-2}u + \frac{n-2}{n-1}\int \sec^{n-2}u\,du$
49. $\int \csc^n u\,du = \frac{-1}{n-1}\cot u\csc^{n-2}u + \frac{n-2}{n-1}\int \csc^{n-2}u\,du$
50. $\int \sin au\sin bu\,du = \frac{\sin(a-b)u}{2(a-b)} - \frac{\sin(a+b)u}{2(a+b)} + C$
51. $\int \cos au\cos bu\,du = \frac{\sin(a-b)u}{2(a-b)} + \frac{\sin(a+b)u}{2(a+b)} + C$
52. $\int \sin au\cos bu\,du = -\frac{\cos(a-b)u}{2(a-b)} - \frac{\cos(a+b)u}{2(a+b)} + C$
53. $\int u\sin u\,du = \sin u - u\cos u + C$
54. $\int u\cos u\,du = \cos u + u\sin u + C$
55. $\int u^n\sin u\,du = -u^n\cos u + n\int u^{n-1}\cos u\,du$
56. $\int u^n\cos u\,du = u^n\sin u - n\int u^{n-1}\sin u\,du$
57. $\int \sin^n u\cos^m u\,du$
$= -\frac{\sin^{n-1}u\cos^{m+1}u}{n+m} + \frac{n-1}{n+m}\int \sin^{n-2}u\cos^m u\,du$
$= \frac{\sin^{n+1}u\cos^{m-1}u}{n+m} + \frac{m-1}{n+m}\int \sin^n u\cos^{m-2}u\,du$

Inverse Trigonometric Forms

58. $\int \sin^{-1}u\,du = u\sin^{-1}u + \sqrt{1-u^2} + C$
59. $\int \cos^{-1}u\,du = u\cos^{-1}u - \sqrt{1-u^2} + C$
60. $\int \tan^{-1}u\,du = u\tan^{-1}u - \frac{1}{2}\ln(1+u^2) + C$
61. $\int u\sin^{-1}u\,du = \frac{2u^2-1}{4}\sin^{-1}u + \frac{u\sqrt{1-u^2}}{4} + C$
62. $\int u\cos^{-1}u\,du = \frac{2u^2-1}{4}\cos^{-1}u - \frac{u\sqrt{1-u^2}}{4} + C$
63. $\int u\tan^{-1}u\,du = \frac{u^2+1}{2}\tan^{-1}u - \frac{u}{2} + C$
64. $\int u^n\sin^{-1}u\,du = \frac{1}{n+1}\left[u^{n+1}\sin^{-1}u - \int\frac{u^{n+1}\,du}{\sqrt{1-u^2}}\right],\quad n\neq -1$
65. $\int u^n\cos^{-1}u\,du = \frac{1}{n+1}\left[u^{n+1}\cos^{-1}u + \int\frac{u^{n+1}\,du}{\sqrt{1-u^2}}\right],\quad n\neq -1$
66. $\int u^n\tan^{-1}u\,du = \frac{1}{n+1}\left[u^{n+1}\tan^{-1}u - \int\frac{u^{n+1}\,du}{1+u^2}\right],\quad n\neq -1$

Forms Involving $\sqrt{a^2-u^2}$, $a>0$

67. $\int \sqrt{a^2-u^2}\,du = \frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\sin^{-1}\frac{u}{a} + C$
68. $\int u^2\sqrt{a^2-u^2}\,du = \frac{u}{8}(2u^2-a^2)\sqrt{a^2-u^2} + \frac{a^4}{8}\sin^{-1}\frac{u}{a} + C$
69. $\int \frac{\sqrt{a^2-u^2}}{u}\,du = \sqrt{a^2-u^2} - a\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right| + C$
70. $\int \frac{\sqrt{a^2-u^2}}{u^2}\,du = -\frac{1}{u}\sqrt{a^2-u^2} - \sin^{-1}\frac{u}{a} + C$
71. $\int \frac{u^2\,du}{\sqrt{a^2-u^2}} = -\frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\sin^{-1}\frac{u}{a} + C$
72. $\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right| + C$
73. $\int \frac{du}{u^2\sqrt{a^2-u^2}} = -\frac{1}{a^2u}\sqrt{a^2-u^2} + C$
74. $\int (a^2-u^2)^{3/2}\,du = -\frac{u}{8}(2u^2-5a^2)\sqrt{a^2-u^2} + \frac{3a^4}{8}\sin^{-1}\frac{u}{a} +$
75. $\int \frac{du}{(a^2-u^2)^{3/2}} = \frac{u}{a^2\sqrt{a^2-u^2}} + C$

Forms Involving $\sqrt{u^2-a^2}$, $a>0$

76. $\int \sqrt{u^2-a^2}\,du = \frac{u}{2}\sqrt{u^2-a^2} - \frac{a^2}{2}\ln\left|u+\sqrt{u^2-a^2}\right| + C$
77. $\int u^2\sqrt{u^2-a^2}\,du$
$= \frac{u}{8}(2u^2-a^2)\sqrt{u^2-a^2} - \frac{a^4}{8}\ln\left|u+\sqrt{u^2-a^2}\right| + C$
78. $\int \frac{\sqrt{u^2-a^2}}{u}\,du = \sqrt{u^2-a^2} - a\cos^{-1}\frac{a}{|u|} + C$
79. $\int \frac{\sqrt{u^2-a^2}}{u^2}\,du = -\frac{\sqrt{u^2-a^2}}{u} + \ln\left|u+\sqrt{u^2-a^2}\right| + C$
80. $\int \frac{du}{\sqrt{u^2-a^2}} = \ln\left|u+\sqrt{u^2-a^2}\right| + C$
81. $\int \frac{u^2\,du}{\sqrt{u^2-a^2}} = \frac{u}{2}\sqrt{u^2-a^2} + \frac{a^2}{2}\ln\left|u+\sqrt{u^2-a^2}\right| + C$
82. $\int \frac{du}{u^2\sqrt{u^2-a^2}} = \frac{\sqrt{u^2-a^2}}{a^2u} + C$
83. $\int \frac{du}{(u^2-a^2)^{3/2}} = -\frac{u}{a^2\sqrt{u^2-a^2}} + C$

Forms Involving $\sqrt{a^2+u^2}$, $a>0$

84. $\int \sqrt{a^2+u^2}\,du = \frac{u}{2}\sqrt{a^2+u^2} + \frac{a^2}{2}\ln(u+\sqrt{a^2+u^2}) + C$
85. $\int u^2\sqrt{a^2+u^2}\,du$
$= \frac{u}{8}(a^2+2u^2)\sqrt{a^2+u^2} - \frac{a^4}{8}\ln(u+\sqrt{a^2+u^2}) + C$
86. $\int \frac{\sqrt{a^2+u^2}}{u}\,du = \sqrt{a^2+u^2} - a\ln\left|\frac{a+\sqrt{a^2+u^2}}{u}\right| + C$
87. $\int \frac{\sqrt{a^2+u^2}}{u^2}\,du = -\frac{\sqrt{a^2+u^2}}{u} + \ln(u+\sqrt{a^2+u^2}) + C$
88. $\int \frac{du}{\sqrt{a^2+u^2}} = \ln(u+\sqrt{a^2+u^2}) + C$
89. $\int \frac{u^2\,du}{\sqrt{a^2+u^2}} = \frac{u}{2}\sqrt{a^2+u^2} - \frac{a^2}{2}\ln(u+\sqrt{a^2+u^2}) + C$
90. $\int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a}\ln\left|\frac{\sqrt{a^2+u^2}+a}{u}\right| + C$
91. $\int \frac{du}{u^2\sqrt{a^2+u^2}} = -\frac{\sqrt{a^2+u^2}}{a^2u} + C$
92. $\int \frac{du}{(a^2+u^2)^{3/2}} = \frac{u}{a^2\sqrt{a^2+u^2}} + C$

Forms Involving $a+bu$

93. $\int \frac{u\,du}{a+bu} = \frac{1}{b^2}(a+bu-a\ln|a+bu|) + C$
94. $\int \frac{u^2\,du}{a+bu} = \frac{1}{2b^3}[(a+bu)^2 - 4a(a+bu) + 2a^2\ln|a+bu|] + C$
95. $\int \frac{du}{u(a+bu)} = \frac{1}{a}\ln\left|\frac{u}{a+bu}\right| + C$
96. $\int \frac{du}{u^2(a+bu)} = -\frac{1}{au} + \frac{b}{a^2}\ln\left|\frac{a+bu}{u}\right| + C$
97. $\int \frac{u\,du}{(a+bu)^2} = \frac{a}{b^2(a+bu)} + \frac{1}{b^2}\ln|a+bu| + C$
98. $\int \frac{du}{u(a+bu)^2} = \frac{1}{a(a+bu)} - \frac{1}{a^2}\ln\left|\frac{a+bu}{u}\right| + C$
99. $\int \frac{u^2\,du}{(a+bu)^2} = \frac{1}{b^3}\left(a+bu-\frac{a^2}{a+bu} - 2a\ln|a+bu|\right) + C$
100. $\int u\sqrt{a+bu}\,du = \frac{2}{15b^2}(3bu-2a)(a+bu)^{3/2} + C$
101. $\int u^n\sqrt{a+bu}\,du$
$= \frac{2}{b(2n+3)}\left[u^n(a+bu)^{3/2} - na\int u^{n-1}\sqrt{a+bu}\,du\right]$
102. $\int \frac{u\,du}{\sqrt{a+bu}} = \frac{2}{3b^2}(bu-2a)\sqrt{a+bu} + C$
103. $\int \frac{u^n\,du}{\sqrt{a+bu}} = \frac{2u^n\sqrt{a+bu}}{b(2n+1)} - \frac{2na}{b(2n+1)}\int\frac{u^{n-1}\,du}{\sqrt{a+bu}}$
104. $\int \frac{du}{u\sqrt{a+bu}} = \frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{a+bu}-\sqrt{a}}{\sqrt{a+bu}+\sqrt{a}}\right| + C,\quad \text{if } a>0$
$= \frac{2}{\sqrt{-a}}\tan^{-1}\sqrt{\frac{a+bu}{-a}} + C,\quad \text{if } a<0$
105. $\int \frac{du}{u^n\sqrt{a+bu}} = -\frac{\sqrt{a+bu}}{a(n-1)u^{n-1}} - \frac{b(2n-3)}{2a(n-1)}\int\frac{du}{u^{n-1}\sqrt{a+bu}}$
106. $\int \frac{\sqrt{a+bu}}{u}\,du = 2\sqrt{a+bu} + a\int\frac{du}{u\sqrt{a+bu}}$
107. $\int \frac{\sqrt{a+bu}}{u^2}\,du = -\frac{\sqrt{a+bu}}{u} + \frac{b}{2}\int\frac{du}{u\sqrt{a+bu}}$

Forms Involving $\sqrt{2au-u^2}$, $a>0$

108. $\int \sqrt{2au-u^2}\,du = \frac{u-a}{2}\sqrt{2au-u^2} + \frac{a^2}{2}\cos^{-1}\left(\frac{a-u}{a}\right) + C$
109. $\int u\sqrt{2au-u^2}\,du$
$= \frac{2u^2-au-3a^2}{6}\sqrt{2au-u^2} + \frac{a^3}{2}\cos^{-1}\left(\frac{a-u}{a}\right) + C$
110. $\int \frac{du}{\sqrt{2au-u^2}} = \cos^{-1}\left(\frac{a-u}{a}\right) + C$
111. $\int \frac{du}{u\sqrt{2au-u^2}} = -\frac{\sqrt{2au-u^2}}{au} + C$

1. (**45 points**)Indicate whether the following statements are true or false. ("True" means "Always true", "false" means "sometimes false".) No justification is necessary on this problem. **Write the entire word "True" or "False"**. Illegible or abbreviated answers will receive no credit.

   In these statements, $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$ are vectors, $\mathbf{u}$ is a unit vector, a, b, c, d, and L are real numbers, t is a parameter, x, y, z, r, θ, ρ, and ϕ are the usual rectangular, polar, or spherical coordinates for $\mathbb{R}^3$, $C : \mathbf{r}(t) = \langle x(t), y(t), z(t)\rangle$ is a smooth curve in $\mathbb{R}^3$, $F = F(x, y, z)$ is a smooth function on $\mathbb{R}^3$, f and g are smooth functions on their domains, which are subsets of $\mathbb{R}^2$ or $\mathbb{R}^3$, and R is a closed and bounded region contained in the domains of f and g. dA and dV mean what they mean in the text.

   (a) $\iint\limits_R f(x,y) + g(x,y)dA = \iint\limits_R f(x,y)dA + \iint\limits_R g(x,y)dA.$

   (b) $\frac{\partial}{\partial x}(f+g) = \frac{\partial f}{\partial x} + \frac{\partial g}{\partial x}.$

   (c) The vector $\langle dx, dy, dz\rangle$ is normal to the graph of $z = f(x, y)$.

   (d) If $\lim_{x\to 0} f(x,0) = \lim_{y\to 0} f(0,y) = L$, then $\lim_{(x,y)\to(0,0)} f(x,y) = L$.

   (e) $\frac{d\mathbf{r}}{dt} = \left\langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \right\rangle.$

   (f) The lines $\langle x, y, z\rangle = \langle -2, 3, 2\rangle + \langle 3, 4, -4\rangle\, t$ and $\langle x, y, z\rangle = \langle 0, -1, -4\rangle + \langle -5, 0, -3\rangle\, t$ intersect at the point $(-5, -1, 6)$.

   (g) If $f(x, y) = x + y$, then $|D_{\mathbf{u}}(f)(x, y)| \le 2$ for all x and y.

   (h) $\frac{\partial}{\partial x}(fg) = \frac{\partial f}{\partial x}\frac{\partial g}{\partial x}.$

   (i) $\|\mathbf{x}\| + \|\mathbf{y}\| = \|\mathbf{x} + \mathbf{y}\|.$

(j) If z is defined implicitly as a function of x and y by $F(x, y, z) = 0$, then

$$\frac{\partial z}{\partial x} = \frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}}.$$

(k) If S is the sphere of radius one about the origin, then $\iiint\limits_S f dV =$

$$\int_{\phi=0}^{\phi=2\pi} \int_{\theta=0}^{\theta=2\pi} \int_{\rho=0}^{\rho=1} f\rho^2 \sin\phi \, d\rho d\theta d\phi.$$

(l) $(\mathbf{x} \times \mathbf{y}) \times \mathbf{z} = \mathbf{x} \times (\mathbf{y} \times \mathbf{z})$.

(m) $\displaystyle\int_a^b \int_c^d f(x, y) dx dy = \int_a^b \int_c^d f(x, y) dy dx.$

(n) If $f(x, y) \geq 0$ for all $(x, y) \in R$, then $\iint\limits_R f dA \geq 0$.

(o) If (a, b) is a critical point of f, $f_{xx}(a, b) = 1$, and $f_{yy}(a, b) = -1$, then (a, b) is a saddle point of f.

2. (**10 points**) Let $\mathbf{x} = \langle -3, 0, 2\rangle$ and $\mathbf{y} = \langle 0, -2, 2\rangle$.

(a) Find $\mathbf{x} \bullet \mathbf{y}$.

(b) Find $\|\mathbf{x}\|$.

3. (**25 points**) Let $P = (-1, 3, 0)$, $Q = (-1, 1, -2)$, $R = (4, -2, 0)$, and $S = (4, 0, 2)$.

(a) Verify that $PQRS$ is a parallelogram.

(b) Find the area of $PQRS$. (We have seen at least three ways to answer this question in class. Most of you will not use an integral; you should state any relevant formulas before using them, and your answer should be a number. If you use double integrals, you may leave your answer in terms of iterated integrals. If you use single integrals, you may leave your answer as an integral or integrals, provided that you define the integrand and the limits of integration unambiguously, and write a sentence explaining why your answer is correct.)

(c) Find an equation for the plane containing $PQRS$.

4. (**35 points**) Consider the curve $C : \mathbf{r}(t) = \big(t, t^2, t^3\big)$.

   (a) Find a parametrization of the tangent line to C at the point $P = (-1, 1, -1)$.

   (b) Find the arc length of C from $P = (-1, 1, -1)$ to $Q = (2, 4, 8)$. (Your answer should be a definite integral. Simplify the integrand and the limits of integration as much as you can, but **do not evaluate**.)

5. (**15 points**) Find $\frac{dy}{dx}$, if

$$y = \left(3x^5 - 7x^2 + 4\right)^3 - \left(3x^5 - 7x^2 + 4\right)^2 \left(\frac{x^4 + \sqrt{x}}{x^2}\right) + \left(3x^5 - 7x^2 + 4\right)\left(\frac{x^4 + \sqrt{x}}{x^2}\right)^2 - \left(\frac{x^4 + \sqrt{x}}{x^2}\right)^3 + \frac{3x^5 - 7x^2 + 4}{\frac{x^4 + \sqrt{x}}{x^2}}.$$

(If you use a technique from multivariable calculus, you may leave your answer in any form that an Algebra II student would understand. If you use only techniques from Calculus I, you must simplify your answer completely.)

6. (**25 points**) Consider the function $F(x, y, z) = xe^{-yz}$.

   (a) Compute ∇F.

   (b) Find an equation for the tangent plane to the level surface $F(x, y, z) = 1$ at the point $(1, 2, 0)$.

7. (**15 points**) Some values of a continuous function $f(x, y)$ on the rectangle $R = \{0 \le x \le 30, 0 \le y \le 20\}$ are given in the table below. (Apparently f is hard to compute, because some values are unknown). Estimate the value of $\iint_R f(x, y)dA$ using a Riemann sum with at least six summands.

   Leave your answer as a Riemann sum.

y \ x	0	5	10	15	20	25	30
0	2	?	4	?	?	?	8
5	2	?	4	?	8	10	?
10	?	?	?	8	?	?	?
15	2	3	?	?	6	8	7
20	2	?	2	2	?	?	4

8. (**40 points**) Express each of the following as an iterated integral in an appropriate coordinate system. If you use coordinates other than rectangular, polar, cylindrical, or spherical, you must define them. **Do not evaluate**.

   (a) $\iiint_R x^2 + y^2 + z^2 \, dxdydz$, where R is the region defined by the inequalities $\{x \geq 0, z \leq 0, x^2 + y^2 + z^2 \leq 4\}$.

   (b) $\iiint_R y \, dxdydz$, where R is the region defined by the inequalities

   $$x^2 + y^2 \leq z \leq 5 \qquad \text{and} \qquad 0 \leq y \leq 1.$$

9. (**20 points**) Find $\int_C \langle e^x(z+1), -\cos y, e^x\rangle \bullet d\mathbf{s}$, where C is the curve that starts at $(1,0,0)$, moves clockwise around the unit circle to $(0,1,0)$, then moves along the curve $(x,y,z) = (t, 1-t, t^2-t)$ to its ending point at $(1,0,0)$.

10. (**20 points**) Consider the following problem:

> Find the maximum and minimum value of x^2y+x+y, subject to the constraint $xy = 4$.

Assuming that maximum and minimum values exist, write down a system of equations whose solutions will be useful in solving this problem. Then explain how those solutions will help in determining the desired values. **Do not solve the problem.**

11. (**Extra credit: 10 points**) Prove that the optimization problem in #10 has no solution.

12. (**Extra credit: 10 points**) In the space remaining on this page, write the largest integer you can. You do not have to simplify, as, for example, "10^3" may take up less space than "1000". However, I should be able (at least in theory) to determine the precise value of your number with no ambiguity or reference to the experiential universe (so, for example, "the number of stars in the sky" is right out), so you may need to use some space defining your notation.