Math 2163

Jeff Mermin's section, Test 1, September 20

On the essay questions (# 4–10) write legibly in complete sentences, in such a way that I can easily tell what you are doing and why.

Do not evaluate any integrals on this test. If you would take an integral, instead simplify the integrand and the limits of integration (if any), and leave the integral as your final answer.

1. (**30 points**)Indicate whether the following statements are true or false. ("True" means "Always true", "false" means "sometimes false".) No justification is necessary on this problem. **Write the entire word "True" or "False"**. Illegible or abbreviated answers will receive no credit.

   In the statements below, a is a real number, $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are vectors in $\mathbb{R}^3$, t is a parameter, x, y, and z are variables which are each differentiable functions of t, $\mathbf{r} = \mathbf{r}(t) = (x(t), y(t), z(t))$ is a parametrization of a curve, and P is a point in $\mathbb{R}^3$.

   (a) $|\mathbf{v} \bullet \mathbf{w}| \leq \|\mathbf{v}\|\|\mathbf{w}\|$.

   (b) $\mathbf{v} - \mathbf{w} = \mathbf{w} - \mathbf{v}$.

   (c) $\frac{d}{dt}\|\mathbf{r}\| = \left\|\frac{d\mathbf{r}}{dt}\right\|$

   (d) $a(\mathbf{v} + \mathbf{w}) = a\mathbf{v} + a\mathbf{w}$.

   (e) $(\mathbf{u} \bullet \mathbf{v})\mathbf{w} = \mathbf{u}(\mathbf{v} \bullet \mathbf{w})$.

   (f) The equations $(x, y, z) = P + t\mathbf{v}$ and $(x, y, z) = P - t\mathbf{v}$ define the same line.

   (g) $(\mathbf{u} \times \mathbf{v}) \times \mathbf{w} = \mathbf{u} \times (\mathbf{v} \times \mathbf{w})$.

   (h) The equations $x = 2$, $y = -1$, $z = 0$ define a line in $\mathbb{R}^3$.

   (i) $\mathbf{v} \times \mathbf{w} = \mathbf{w} \times \mathbf{v}$.

   (j) $\int_{t=0}^{t=1} \frac{d\mathbf{r}}{dt} dt = \mathbf{r}(1) - \mathbf{r}(0)$.

2. (**20 points**) Let $\mathbf{v}$, $\mathbf{w}$, $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$ be vectors in $\mathbb{R}^3$. Are the following expressions vectors, scalars, or nonsense? (No justification is necessary on this problem, but wrong answers with good explanations may receive credit.)

(a) $(((\mathbf{v} \times \mathbf{w}) - \mathbf{x}) - \mathbf{y}) \times \mathbf{z}$

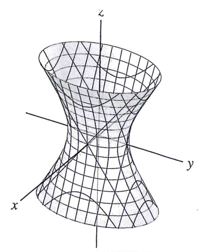
(b) $(\mathbf{v} \bullet ((\mathbf{w} \bullet \mathbf{x})\mathbf{y}))\mathbf{z}$

(c) $(\mathbf{v} \bullet (\mathbf{w} - (\mathbf{x} + \mathbf{y}))) \bullet \mathbf{z}$

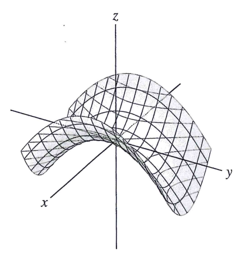
(d) $(\mathbf{v} \bullet \mathbf{w})(\mathbf{x} \bullet (\mathbf{y} \times \mathbf{z}))$

3. (**20 points**) For each of the quadric surfaces below, identify it by type (ellipsoid, hyperbolic cylinder, etc.) **or** suggest a possible equation. (No justification is necessary on this problem, but wrong answers with good explanations may receive credit.)

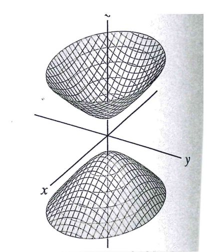
(a)



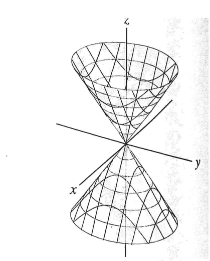
(b)



(c)



(d)



4. (**20 points**) Let $\mathbf{x} = \langle -2, -2, 4 \rangle$ and $\mathbf{y} = \langle 0, 3, -1 \rangle$. Compute the following:

   (a) $\mathbf{x} - 3\mathbf{y}$.

   (b) $\mathbf{x} \bullet \mathbf{y}$.

   (c) $\mathbf{x} \times \mathbf{y}$.

   (d) $(\mathbf{x} - 3\mathbf{y}) \times \mathbf{y}$.

5. (**10 points**) Find a parametrization of the line passing through the points $P = (0, 1, -1)$ and $Q = (-1, 5, 2)$.

6. (**15 points**) Find an equation for the plane containing the point $R = (-3, 2, 3)$ and perpendicular to the line $(x, y, z) = (2, -1, 0) + (2, 3, 3)t$.

7. (**15 points**) Find a parametrization of the line of intersection of the planes $G : x - 3y + 4z = 1$ and $H : 3x + 2y - 2z = -6$.

8. (**20 points**) Let C be the curve defined by the parametrization $\mathbf{r}(t) = \big(e^t, t \ln t, t^2 - t\big)$. Find a parametrization of the tangent line to C at the point $P = (e, 0, 0)$.

9. (**Extra credit: 10 points**) Let S be the surface defined by the equation (in spherical coordinates) $\rho = \cos\theta \sin\phi$. Find (and simplify, if necessary) an equation for S in the form $F(x, y, z) = 0$, and describe S verbally.

10. (**Extra credit: 10 points**) Let F be the plane defined by the equation $2x - y + 2z = 4$. Find another plane G which is parallel to F but exactly two units away.