

Math 2163

Jeff Mermin's sections, Test 3, December 2

On the essay questions (# 2-6) write legibly in complete sentences, in such a way that I can easily tell what you are doing and why.

1. (30 points) Indicate whether the following statements are true or false. (“True” means “Always true”, “false” means “sometimes false”.) No justification is necessary on this problem. **Write the entire word “True” or “False”**. Illegible or abbreviated answers will receive no credit.

In the statements below, \mathbf{a} , \mathbf{b} , and \mathbf{c} are vectors in \mathbb{R}^3 , a , b , c , and d are real numbers, $C : \mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$ is a curve in \mathbb{R}^3 with associated vectors \mathbf{T} , \mathbf{N} , and \mathbf{B} , R is a closed and bounded region in \mathbf{R}^2 or \mathbf{R}^3 , and f is a function on R with continuous partial derivatives of all orders.

(a) The area of R is $\iint_R 1 dA$.

(b) $(\mathbf{a} + \mathbf{b}) + \mathbf{c} = \mathbf{a} + (\mathbf{b} + \mathbf{c})$.

(c) $\mathbf{a} \times \mathbf{b} = \mathbf{b} \times \mathbf{a}$.

(d) $\mathbf{N} = \mathbf{B} \times \mathbf{T}$.

(e) If (a, b) is a critical point of f , $f_{xx}(a, b) = 1$, and $f_{yy}(a, b) = -1$, then (a, b) is a saddle point of f .

(f) $\mathbf{a} - \mathbf{b} = \mathbf{b} - \mathbf{a}$.

(g) If $f(x, y) \geq 0$ for all $(x, y) \in R$, then $\iint_R f dA \geq 0$.

(h) The distance between the planes $F : ax + by + cz = d$ and $G : ax + by + cz = d'$ is $|d - d'|$.

(i) If $R = \{(x, y, z) : 1 \leq x^2 + y^2 + z^2 \leq 16\}$, then $\iiint_R dV = 84\pi$.

(j) $\frac{d\mathbf{r}}{dt} = \left\langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \right\rangle$.

2. (30 points) Evaluate the integrals:

(a)
$$\int_{x=1}^{x=4} \int_{y=0}^{y=2} 4x^3 + 3y^2 \, dy \, dx$$

(b)
$$\int_{x=0}^{x=4} \int_{y=0}^{y=x} \sqrt{xy} \, dy \, dx$$

3. (20 points) Consider the iterated integral $I = \int_{y=0}^{y=\ln 3} \int_{x=e^y}^{x=3} \frac{1}{\ln x} dx dy$.

(a) Reverse the order of integration.

(b) Evaluate I , using whichever order seems easier.

4. (10 points) Some values of a continuous function $f(x, y)$ on the rectangle $R = \{0 \leq x \leq 12, 10 \leq y \leq 30\}$ are given in the table below. (Apparently f is hard to compute, because some values are unknown). Estimate the value of $\iint_R f(x, y) dA$ using a Riemann sum with at least six summands.

Warning: The $x = 8$ column is missing.

		x					
		0	2	4	6	10	12
y	10	?	-10	-10	-9	-6	0
	15	-7	-4	?	2	4	8
	20	-1	3	?	8	?	11
	25	-3	-1	-1	-1	0	?
	30	7	?	10	?	11	13

5. **(60 points)** Express the following as iterated integrals, using a coordinate system of your choice.

(a) $\iint_R x + y \, dA$, where R is the region between the curves $y = x^2$ and $y = x^3$.

- (b) The mass of the hemisphere $R = \{(x, y, z) : x^2 + y^2 + z^2 \leq 9, z \geq 0\}$, if its density at the point (x, y, z) is $25 - x^2 - y^2$.

6. $\iint_R \frac{x+2y}{x-2y} dA$, where R is the region $\{(x, y) : |x| + |2y| \leq 4\}$.

7. **(Extra credit: 20 points)** The *standard normal density* or *bell curve* is very important in probability and statistics, where it is used to describe certain kinds of repeatable experiments. The bell curve is given by the function $f(x) = \frac{1}{A}e^{-\frac{x^2}{2}}$, where A is chosen to make the bell curve into a probability density, that is, $\int_{x=-\infty}^{x=\infty} f(x) dx = 1$.

Thus $A = \int_{x=-\infty}^{x=\infty} e^{-\frac{x^2}{2}} dx$. Unfortunately, it is known that the indefinite integral $\int e^{-\frac{x^2}{2}} dx$ cannot be evaluated algebraically. However, we can still solve for the exact value of A .

$$\text{Let } B = \int_{x=-\infty}^{x=\infty} \int_{y=-\infty}^{y=\infty} e^{-\frac{x^2}{2}} e^{-\frac{y^2}{2}} dy dx.$$

- (a) Describe the relationship between A and B .

- (b) Rewrite B using polar coordinates.

- (c) Find A .