Math 2163

Jeff Mermin's sections, Test 2, November 2

On the essay questions (# 2–8) write legibly in complete sentences, in such a way that I can easily tell what you are doing and why.

1. (**30 points**)Indicate whether the following statements are true or false. ("True" means "Always true", "false" means "sometimes false".) No justification is necessary on this problem. **Write the entire word "True" or "False"**. Illegible or abbreviated answers will receive no credit.

   In the statements below, $C : \mathbf{r}(t) = \langle x(t), y(t), z(t)\rangle$ is a curve, (a, b) is a point in the closed and bounded region D, $f(x, y)$ is a continuous function defined on some region including D, $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are vectors, $\mathbf{u}$ is a unit vector, and L is a number.

   (a) $\frac{d\mathbf{r}}{dt} = \left\langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \right\rangle$.

   (b) If $\lim_{x\to 0} f(x, 0) = \lim_{y\to 0} f(0, y) = L$, then $\lim_{(x,y)\to(0,0)} f(x, y) = L$.

   (c) $\mathbf{a} \bullet (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \bullet \mathbf{b})(\mathbf{a} \bullet \mathbf{c})$.

   (d) The angle between two planes is equal to the angle between their normal vectors.

   (e) $(\mathbf{a} + \mathbf{b}) + \mathbf{c} = \mathbf{a} + (\mathbf{b} + \mathbf{c})$.

   (f) If $\lim_{(x,y)\to(a,b)} f(x, y) = L$, then $\lim_{(x,y)\to(a,b)} \cos f(x, y) = \cos L$.

   (g) If f has an absolute maximum on the region D at (a, b), then f has a local maximum at (a, b).

   (h) $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} = \mathbf{a} \times (\mathbf{b} \times \mathbf{c})$.

   (i) Two lines define a plane.

   (j) If $f(x, y) = x + y$, then $|D_{\mathbf{u}}(f)(x, y)| \leq 2$ for all x and y.

2. (**15 points**) Prove that the limit

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{x^2+y}$$

does not exist.

3. (**15 points**) Find an equation for the tangent plane to the graph of the function $z = \frac{x+y}{4x}$ at the point $(1, 3, 1)$.

4. (**30 points**) Compute all the second partial derivatives of

$$f(x, y) = \sin(x)\cos(y) - \cos(x)\sin(y).$$

5. (**15 points**) The surface of a mountain has equation $z = \ln(1 + 3x^2y^3)$. A bucket of water is emptied above the point $(2, 1)$. In which direction does the water flow? (You may give only the horizontal direction, as a cartographer would.)

6. (**15 points**) Find $\frac{dz}{dt}$, if $z = \sqrt{\left(\frac{\ln t}{7}\right)^4 + 3\left(\frac{\ln t}{7}\right)\left(\frac{t^3}{e^t}\right) + \left(\frac{t^3}{e^t}\right)^4} - 2.$

7. (**30 points**) Consider the function $f(x, y) = 9xy - x^3 - y^3 - 6$. You do not need to compute the partial derivatives:

$$\begin{aligned} f_x &= 9y - 3x^2 \\ f_y &= 9x - 3y^2 \\ f_{xx} &= -6x \\ f_{xy} &= 9 \\ f_{yy} &= -6y. \end{aligned}$$

Determine whether or not the points below are critical points. If they are critical points, determine whether they are saddle points, local maxima, or local minima.

(a) $P = (0, 0)$.

(b) $Q = (0, 3)$.

(c) $R = (3, 0)$.

(d) $S = (3, 3)$.

8. (**Extra credit: 20 points**) Find the maximum and minimum values of the function $f(x, y) = x^2 + x^2y + y^3$ on the region $x^2 + y^2 \leq 1$. (This region has an interior.)