 Name:____________________________

Math 2163

Jeff Mermin's sections, Test 2, October 23

On the essay questions (# 2–7) write legibly in complete sentences, in such a way that I can easily tell what you are doing and why.

1. (**30 points**)Indicate whether the following statements are true or false. ("True" means "Always true", "false" means "sometimes false".) No justification is necessary on this problem. **Write the entire word "True" or "False"**. Illegible or abbreviated answers will receive no credit.

   In the statements below, a, b, c, d, d', and L are numbers, $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are vectors, $f(x,y)$ and $F(x,y,z)$ are smooth functions in the sense that all their partial derivatives are defined and continuous on their domains, and $C : \mathbf{r}(t)$ is a curve with associated vectors $\mathbf{T}$, $\mathbf{N}$, and $\mathbf{B}$.

   (a) If (a,b) is a critical point of f, $f_{xx}(a,b) = 1$, and $f_{yy}(a,b) = -1$, then (a,b) is a saddle point of f.

   (b) If $\lim_{(x,y)\to(a,b)} f(x,y) = L$, then $\lim_{(x,y)\to(a,b)} (\cos f(x,y)) = \cos L$.

   (c) If $\lim_{(x,y)\to(a,b)} f(x,y) = L$, then $\lim_{x\to a} f(x,b) = L$.

   (d) The distance between the planes $F : ax + by + cz = d$ and $G : ax + by + cz = d'$ is $|d - d'|$.

   (e) $\mathbf{a} \bullet (\mathbf{b} + \mathbf{c}) = \mathbf{a} \bullet \mathbf{b} + \mathbf{a} \bullet \mathbf{c}$.

   (f) $\mathbf{B} = \mathbf{T} \times \mathbf{N}$.

   (g) If z is defined implicitly as a function of x and y by $F(x,y,z) = 0$, then
   $$\frac{\partial z}{\partial x} = \frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}}.$$

   (h) There are functions $h_1(x)$ and $h_2(y)$ such that $f(x,y) = h_1(x) + h_2(y)$.

   (i) $\mathbf{a} \times \mathbf{b} = \mathbf{b} \times \mathbf{a}$.

   (j) There is a function $g(x,y)$ such that $g_x(x,y) = x^2 + y^2$ and $g_y(x,y) = x^2 - y^2$.

2. (**30 points**) Compute all the second partial derivatives of $f(x,y) = \frac{x^2-y^2}{xy}$.

3. (**20 points**) Suppose that y is defined as a function of x by the relationship $x \sin y + y \cos x = 0$. Find $\frac{dy}{dx}$.

4. (**20 points**) Find an equation for the tangent plane to the hyperboloid $x^2 + y^2 - z^2 = 4$ at the point $(2, 1, 1)$.

5. (**30 points**) Consider the function $f(x, y) = x^4 - 2x^2 + 3y^2$. You do not need to compute the partial derivatives:

$$\begin{aligned} f_x &= 4x^3 - 4x \\ f_y &= 6y \\ f_{xx} &= 12x^2 - 4 \\ f_{xy} &= 0 \\ f_{yy} &= 6. \end{aligned}$$

Determine whether or not the points below are critical points. If they are critical points, determine whether they are saddle points, local maxima, or local minima.

(a) $P = (0, 0)$.

(b) $Q = (0, 1)$.

(c) $R = (1, 0)$.

6. (**20 points**) Using any appropriate method, find the absolute minimum value of the function $f(x, y) = x^2 + y^2$ subject to the constraint $xy = 3$. (You may assume such a minimum exists.)

7. (**Extra credit: 20 points**) Name, and write down possible equations for, each of the surfaces pictured below.

(a)

(b)

(c)

(d)

(e)