Math 2163

Jeff Mermin's sections, Test 1, September 18

On the essay questions (# 2–8) write legibly in complete sentences, in such a way that I can easily tell what you are doing and why.

Do not evaluate any integrals on this test. If you would take an integral, instead simplify the integrand and the limits of integration (if any), and leave the integral as your final answer.

1. (**30 points**)Indicate whether the following statements are true or false. ("True" means "Always true", "false" means "sometimes false".) No justification is necessary on this problem. **Write the entire word "True" or "False"**. Illegible or abbreviated answers will receive no credit.

   In the statements below, x, y, z, and t are variables, a, b, c, d, d', and L are numbers, $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are vectors, $C : \mathbf{r}(t) = \langle x(t), y(t), z(t)\rangle$ is a curve in space with associated $\mathbf{T}$, $\mathbf{N}$, $\mathbf{B}$, and κ, and $f(x, y)$ is a function.

   (a) $\mathbf{a} \bullet (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \bullet \mathbf{b})(\mathbf{a} \bullet \mathbf{c})$.

   (b) $\mathbf{a} \bullet (\mathbf{b} + \mathbf{c}) = \mathbf{a} \bullet \mathbf{b} + \mathbf{a} \bullet \mathbf{c}$.

   (c) $\frac{d\mathbf{r}}{dt} = \left\langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \right\rangle$.

   (d) There are functions $g(x)$ and $h(y)$ such that $f(x, y) = g(x) + h(y)$.

   (e) $\mathbf{B} = \mathbf{T} \times \mathbf{N}$.

   (f) The distance between the planes $F : ax + by + cz = d$ and $G : ax + by + cz = d'$ is $|d - d'|$.

   (g) If C is a circle, then κ is its radius.

   (h) $|\mathbf{a} \bullet \mathbf{b}| \le |\mathbf{a}||\mathbf{b}|$.

   (i) $\mathbf{a} \times \mathbf{b} = \mathbf{b} \times \mathbf{a}$.

   (j) If $\lim_{(x,y)\to(a,b)} f(x, y) = L$, then $\lim_{x\to a} f(x, b) = L$.

2. (**30 points**) Let $\mathbf{a} = \langle 7, 7, 7 \rangle$ and $\mathbf{b} = \langle 8, 9, -10 \rangle$. Compute the following:

(a) $10\mathbf{a} - 10\mathbf{b}$.

(b) $\mathbf{a} \bullet \mathbf{b}$.

(c) $\mathbf{a} \times \mathbf{b}$.

(d) $(\mathbf{a} - \mathbf{b}) \times \mathbf{b}$.

3. (**15 points each**) Show that the limits do not exist, or give strong evidence that they do.

   (a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 - y^2}{x^2 + y^2}.$

   (b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{3x^2 y}{x^2 + 2y^2}.$

4. (**20 points**) Find the equation of the plane containing the points $(-10, 10, -6)$, $(2, 0, 10)$, and $(-9, -5, -3)$.

5. (**15 points**) Determine whether the planes $F : 2x + -6y - 7z = -6$ and $G : 7x + 5y = 5$ intersect. If they do, find equations for the line of intersection. If they do not, find the distance between F and G.

6. (**15 points**) Find the length of the curve $C : \mathbf{r}(t) = \langle t, t^2, t^3 \rangle$ on the interval $1 \leq t \leq 4$.

7. (**10 points**) Find equations for the tangent line to the curve $C : \mathbf{r}(t) = \left\langle t^2, \frac{t}{t+1}, e^{2t} \right\rangle$ at the point $(0, 0, 1)$.

8. (**Extra credit: 20 points**) Consider the curve $C : \mathbf{r}(t) = \langle e^t \sin t, t^2, e^t \cos t \rangle$. Compute **T**, **N**, and **B** at the point $(0, 0, 1)$, and find an equation for the osculating plane of C at this point.