## Comments and errata for *Partial Differential Equations for Scientists and Engineers* by Stanley J. Farlow.

Here are some specific comments and errors I found:

**Lesson 3**

Instead of numbering the types of BC instead it is good to use the more standard naming, such as Dirichlet (set the value), Neumann (set the derivative), or Robin (mixed). Unfortunately it seems type 1, type 2, type 3 are different for different authors. For example Wikipedia calls Neumann "type 2", and Robin "type 3", while Farlow has the opposite (probably less standard) numbering.

On page 21 the $A$ (area) is missing from Fourier's law and similarly on the next pages. I guess he just assumes it is part of the $k$. But in eq 3.4 on the next page he puts the $A$ in, though he never mentions it is the area of the cross section.

**Lesson 6**

On page 45 in equation 6.5, the exponential should be $e^{-(n\pi\alpha/L)^2 t}$, the division by $L$ is missing.

On page 47, problem 1. The boundary conditions are probably harder than intended. The whole point of lesson 7 is doing exactly such a problem, so having students do it before doesn't seem right. Probably what was meant are different boundary conditions that are simpler. Also the answer in the back is wrong. I would assume that the boundary conditions that were wanted was something like $u(0,t) = 0$, $u(1,t) = 1$, or any other conditions that the solution in the back satisfies.

**Lesson 9**

On page 61, in note number 1. Farlow calls the $-u$ the "convection term" though that's the term due to lateral heat loss. Convection is the next equation. So in note number 1 it should be "lateral heat loss term" and then $e^{-\beta t}$ could be called the "lateral heat loss factor"

**Lessons 10–13**

In chapters on transform, $\xi$ is sometimes used as the frequency variable and sometimes as dummy variable for example in convolution, etc... I think it would be better not to overuse.

**Lesson 13**

At end of the lesson, the inverse Laplace transform is brushed off as just using the tables, while it does not seem as trivial as the book makes it look.

**Lesson 15**

$\alpha^2$ changes to $D$ for whatever reason and $v$ becomes capital $V$. Not sure why the change in notation is useful.

**Lesson 24**

On page 188, the formula for the 2-dimensional solution has an extra factor $\frac{1}{2\pi c}$ in the second term.

**Lesson 27**

The variable name $\tau$ is an odd choice. Before $\tau$ was used as a new time variable but here that isn't true. $s$ is really "the new time variable" in fact in all examples $t = s$. Perhaps we should use $\xi$, this was used before in the convection lesson 13. In fact there $\tau$ was used for the new time.

**Lesson 30**

It seems it would make sense to first talk about the change of coordinates to polar before doing this section or in this section. It seems out of place to do it in the opposite order.

On page 234, the condition on $R(0)$ being finite should be written more carefully, we of course want to avoid both infinities, perhaps $|R(0)| < \infty$.

On page 237 in the figure 30.3, there is one nodal circle drawn in but all the others are not. I suppose all of them should be drawn or none of them.

On page 238, I don't understand how all the frequencies can have the angular phase shift normalized to zero by a single rotation. In fact, it is obviously not possible in general. It's not hard to come up with initial conditions which cannot be just rotated and cannot have a solution of the form given. Of course, we don't ever solve in general, so it is not a problem, but the paragraph does seem misleading. Googling around I found several class notes that have the same issue, so perhaps this all comes from a single source that everybody starts with. Since none of these places actually try to solve a general IBVP, they never realize they are missing something.

**Lesson 31**

Page 246, top of page item 2. in the list. If the Laplacian is zero at a point it doesn't mean that $u$ is equal to the average of $u$ in neighbouring points, it should say "approximately equal". Same in the bottom of the page.

**Lesson 33**

Page 263, the Fourier series representation is given in a different way from previously with respect to $n = 0$ term. In particular $a_0$ is not the same $a_0$ as before, and $b_0$ is redundant. Not an error, but it's inconsistent with what came before. Also in the computation of Poisson integral formula the constant term is pulled out anyway, so there is no advantage to changing notation either.

Bottom of page 265, the $\theta$ in the integration calculation conflicts with the $\theta$ that's in the argument list of $u(r, \theta)$.

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The following comments are from Arpard Fazakas from Amazon.com review of the book. **Some of these may not be correct comments, I have not checked them**, so just because he says something is wrong, doesn't mean it is (similar disclaimer appears in his original post):

Table 13-2: although the separation of variables method is listed as being inapplicable to nonhomogeneous boundary conditions, in fact it can be used to solve Dirichlet problems on a rectangle with one non-homogeneous boundary.

Lesson 32 p. 251: Laplacian in spherical coordinates fourth term should be $\cot(\phi)$, not $\cot(\theta)$.

Lesson 39 p. 320: step 2 of implicit algorithm for heat problem: $u_{11}$ and $u_{16}$ should be zero, not 1, so first and fourth equations equal zero, not 1, and final result is $u_{22}$ and $u_{25}$ are 0.2, not 0.6, and $u_{23}$ and $u_{24}$ are 0.6, not 0.8. These results are closer to the results given by the analytic solution $u = \frac{\pi}{4} \sum_{n \text{ odd}} \frac{\sin(n\pi x)}{n} \exp(-n^2\pi^2 t)$.

Lesson 41 p. 338: step 3, the coefficients of the new canonical form are computed from equations (41.3), not (41.5).

Lesson 44 p. 359: $J(y) = 1.28$, not 0.46.

Lesson 45: p. 369 problem 2: I believe new function $z(t) = (1 - t)y(t)$, not $(1 - x)y(t)$.

Problem 5: $A = .004$, not .06, and $B = .097$, not .04. The values given in the book do not satisfy the boundary condition $u(x, 1) = 0$. The correct values can be calculated from the analytic solution $u(x, y) = \left(\frac{\cosh(\pi y)-1}{\pi^2} - \frac{\cosh(\pi)-1}{\pi^2 \sinh(\pi)} \sinh(\pi y)\right) \sin(\pi x)$.

Lesson 47 p. 385: I think $\gamma = \frac{t}{(x-t)^2+y^2}$, not $\frac{2t}{\dots}$. This gives results for $u^2 + v^2$ close to those listed in (47.6), whereas using the result for gamma given in the book gives $u^2 + v^2 = 3.95$ and 23.9. Page 386: $\phi(u, v)$ and $\phi(x, y) = 0.53 \ln(u^2 + v^2) + 1$, not 0.57 ln etc.

Answers to Problems:

8.1: $u(x, t) = \frac{4}{\pi} e^{\frac{1}{2}(x-t/2)}$... etc, not $\frac{4}{\pi} e^{-\frac{1}{2}(x-t/2)}$... etc. Also in the sum there should be a term $\exp(-n^2\pi^2 t)$.

9.3: sum should be from $n = 1$ to infinity, not $n = 0$ to infinity.

9.5: $T_n(t) = (-1)^{n+1}$... etc, not $(-1)^n$.

12.3: denominator should be $\sqrt{4\alpha^2 t + 1}$, not $\sqrt{4\alpha^2 + 1}$.

13.3: $\alpha$ should be 1.

20.5: both terms should include $8h$, not $4h$.

24.2: given solution doesn't satisfy initial conditions. I believe $u(x,t)$ should be $\frac{1}{2}((x+ct)+(x-ct))$.

25.2: the exponents of $e$ should be minus and plus $(n^2\pi^2\alpha^2 - b)t$, respectively, not minus and plus $(n^2\pi^2\alpha^2)t$.

25.6: second equation should equal $6\pi + 1$ for $n = 3$, not $8\pi + 1$.

28.4: log term for $u(x,t) = \ln|1 - t/x|$, not $-\ln(t+1)$.

35.5: calculation for a subscript $n$ can be taken further to get $(-1)^{(n-1)/2}\frac{2n+1}{2^n}$ for $n$ odd, zero for $n$ even.

37.3: $u_{i,j} = \frac{1}{4}$ (etc etc) not $\frac{1}{2}$ (etc etc).

37.4: denominator is $2(h^2 - 2)$, not $2(h-2)$.

39.2: $u_{i,1} = 1$, not zero.

41.3: I got $u_{\xi\xi} + u_{\nu\nu} + \left(\frac{\nu^2}{2\sqrt{2}}\right) u_\nu = \frac{1}{2}\exp(-\nu^2/4)$, but this is so different from the book that it may be my bad.

45.2: should be $(z'/(1-x) + z/(1-x)^2)^2$, not $z'/(1-x) + z/(1-x)^2$.

Appendix 3: 3-d spherical Laplacian all thetas should be phi's and vice versa.