A couple of extra questions

Here's a couple of extra problems to try. This is not a complete coverage of the material. You should go over the past exams, the online homeworks, and the exercises in the book.

1. For what a is $f(x, y) = 2x^2 + ay^2 + xy$ is a Harmonic function (solves the Laplace equation)?
2. Compute $d(x^2zy\, dx \wedge dy)$
3. Compute $(x^2\, dx + 4y\, dz) \wedge (5\, dx \wedge dy + 8\, dz \wedge dx + e^z\, dy \wedge dz)$
4. Suppose $(f_1\hat{\imath} + f_2\hat{\jmath} + f_3\hat{k}) \times (g_1\hat{\imath} + g_2\hat{\jmath} + g_3\hat{k}) = z\hat{\imath} + e^y\hat{\jmath} + x^2\hat{k}$. Compute $(f_1\, dx + f_2\, dy + f_3\, dz) \wedge (g_1\, dx + g_2\, dy + g_3\, dz) =$
5. If $d\omega = 3\, dx \wedge dy \wedge dz$ for a certain two-form ω, and S is the boundary of a region R in three-space where volume of R is 2 (and S is oriented with the outer unit normal). Find
$$\int_S \omega$$
6. Compute
$$\int_S 5\, dx \wedge dy + 8\, dz \wedge dx + e^z\, dy \wedge dz$$
for a surface S defined by $z = x^2 + y$, $-1 < x < 1$, $-1 < y < 1$.
7. Find and classify the critical points of
a) $f(x, y) = xy - 2y - 3x + 6$
b) $f(x, y) = 3y^2 - xy - 24y + 2x^2 - 4x + 50$
c) $f(x, y) = x^3 + x^2 + y^2 + xy$
d) $f(x, y) = x^4 - y^4$
8. Compute
$$\int_C (x^2 - y^2)\, dx + y^2\, dy$$
for a C being the sides of the unit square, traversing them counterclockwise.

9. Suppose $\nabla \cdot \vec{F} = x$. Compute

$$\iint_S \vec{F} \cdot \hat{n} \, dS$$

for S being the sides of the unit cube, with $\hat{n}$ being the outer unit normal.

10. Suppose

$$\iint_R \vec{F} \cdot \hat{k} \, dA = 3$$

where R is the bottom square side (the one in the xy-plane) of the unit cube. Suppose that $\nabla \cdot \vec{F} = 0$. Find

$$\iint_S \vec{F} \cdot \hat{n} \, dA$$

where $\hat{n}$ is the outer unit normal and S are the 5 remaining sides of the cube.

11. Suppose you estimated $\int_C y \, dx \approx 9.1$ for a closed curve C. What can you say about the inside of C?

12. Let $f(x, y, z) = x^2 + y + e^{xz^2}$. Let C be a path (any path) from $(1, 2, 3)$ to $(-1, 3, 5)$. Can you compute

$$\int_C \nabla f \cdot \hat{T} \, ds$$

and if so what is it? If not explain why.

13. Suppose a force field is $\vec{F} = (2xz + yze^{xy})\hat{\imath} + (xze^{xy})\hat{\jmath} + (x^2 + 2z + e^{xy})\hat{k}$. Compute the potential of this force field (in the sense of physicists), that is find a function ψ such that $\vec{F} = -\nabla\psi$.

14. Suppose that ψ has a single global minimum (and a single critical point in fact) at the point $(1, 2, 3)$ and goes to plus infinity along any path going from the origin (towards infinity) in any direction. Let $\vec{F}$ be a force field given by $\vec{F} = -\nabla\psi$. Suppose a particle is pulled by $\vec{F}$ and starts at the point $(0, 0, 0)$. What will this particle do?