Homework 9
MATH 5293

1. Prove that $f \in H(\mathbb{C})$ has no zeros in $\mathbb{C}$ if and only if $f = e^g$ where $g \in H(\mathbb{C})$.

2. Show that f is an entire function with m zeros at the origin and all other zeros $\{z_k\}_{k=1}^n \subset \mathbb{C} \setminus \{0\}$ if and only if

$$f(z) = z^m e^{g(z)} \prod_{k=1}^{n} \left(1 - \frac{z}{z_k}\right),$$

where g is entire.

3. Prove that the product

$$\prod_{n=1}^{\infty} \left(1 + \frac{z}{n}\right) e^{-z/n}$$

converges absolutely and uniformly on compact subsets of $\mathbb{C}$.