Homework 6
MATH 5293

1. Let $u \in C^2(G)$ be a real valued function defined on an open set $G \subset \mathbb{C}$. Prove that u is subharmonic in G if and only if $\Delta u \geq 0$ in G.
Hint: The latter assumption implies that u satisfies the Maximum Principle.

2. Prove Jensen's Inequality: If $f : [a, b] \to [c, d]$ is integrable, where $[a, b] \subset \mathbb{R}$ and $[c, d] \subset \mathbb{R}$, and if $\psi : [c, d] \to \mathbb{R}$ is convex, then

$$\psi\left(\int_a^b f(x)\frac{dx}{b-a}\right) \leq \int_a^b \psi\left(f(x)\right)\frac{dx}{b-a}.$$

3. Suppose that $u : G \to [c, d]$ is subharmonic on an open set $G \subset \mathbb{C}$, and $\psi : [c, d] \to \mathbb{R}$ is convex and increasing. Use Jensen's Inequality to show that $\psi \circ u$ is subharmonic in G.